The Interplay Between Product Variety and Customer Satisfaction: Theory and Evidence

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Abstract

The optimal number of products to offer consumers is one of the core strategic problems that firms face. This is increasingly so in digital markets where many firms offer a large product variety. In these markets, consumers purchase products repeatedly, making customer satisfaction an important aspect for firm performance. In this paper, we study the interplay between these two variables, product variety and customer satisfaction. First, we provide a game-theoretic model to analyze this interplay and also determine how the optimal product variety depends on the market environment. Second, we empirically test the resulting predictions using data for video games from Steam and for mobile applications from the Google Play Store. Both data sets come with accurate measures of the key variables in our game-theoretic model and additionally contain plausible instrumental variables for empirical identification. We show theoretically that investment in product portfolio size and investment in customer satisfaction are substitutes for a firm. However, between firms, these variables can either be strategic complements or strategic substitutes. We then derive novel predictions on how market conditions determine a firm's product variety, for which we find ample evidence in our empirical analyses: There is (i) a negative relation between product portfolio size and customer satisfaction, (ii) an inverted u-shape relationship between market value and product variety, and (iii) a positive relation between the number of competitors and product variety. Both our theoretical and empirical results are robust to a wide set of robustness checks.

Keywords: Product Variety, Customer Satisfaction, Multi-Product Competition, Video Games, Mobile Applications

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1 Introduction

The choice of the product portfolio size (i.e., the number of offered products) is an important strategic decision of firms in most industries. In many digital markets, such as the markets for video games, mobile applications (apps), or electronic books, this decision has become even more prominent than in traditional markets, as the costs of developing and launching new products are relatively small. Indeed, in many markets for digital goods, firms offer a large variety of products. At the same time, there is considerable variation within and across market segments. For example, in the video game industry—one of the largest entertainment industries worldwide—some publishers (e.g., Electronic Arts) offer hundreds of games across genres whereas other publishers (e.g., Rockstar Games) only have a few titles in their portfolio.[1] Similarly, developers of apps such as '+Home' and 'fancy keyboard' sell more than 400 wallpaper and 100 keyboard theme apps in the Google Play Store, respectively.[2]

These markets are also characterized by repeated and frequent purchase decisions of consumers who usually buy different games or apps over time. Customer satisfaction is therefore an important determinant in these markets, as a consumer's choice whether to buy and/or download a game or an app of the same firm is to large extent driven by whether the previous product has fulfilled or exceeded expectations.[3] Indeed, many customers provide feedback about their experience and how satisfied they are with a current product via online reviews and consumer ratings.

In the strategy literature—as well as in the economics and marketing literature—several papers analyze the effect of a firm's product variety choice on performance. For example, Lancaster (1990) as well as Bayus and Agarwal (2007) demonstrate that a more extensive product line allows a firm to cater to different consumer preferences, thereby increasing demand. Focusing on the supply side, Cottrell and Nault (2004) as well as Nerkar and Roberts (2004) consider potential benefits from economies of scope. Similarly, the literature on customer satisfaction has pointed out that a higher customer satisfaction leads to a competitive advantage for a firm, foremost due to increased repurchasing behavior of customers. Anderson and Sullivan (1993) as well as Bhattacharya et al. (2021) show that this effect is persistent in many different market environments, while Otto et al. (2020) as well as Hult et al. (2022) demonstrate how the effect depends on consumer characteristics.[4]

The existing literature has provided valuable insights on how product variety and con-

[1] See https://en.wikipedia.org/wiki/Video_game_publisher.

[2] See https://www.androidrank.org/developer?id=4803243400814519754 and https://www.androidrank.org/developer?id=%2BHOME+by+Ateam+Entertainment.

[3] See e.g., https://digital.com/54-of-online-shoppers-read-reviews-before-every-purchase/.

[4] We provide a more detailed literature review at the end of this section.

sumer satisfaction can increase firm performance, but has focused on each variable *separately*. However, the *interplay* between the two variables is of equal importance: although both instruments help gaining competitive advantage, a firm has only limited resources and needs to decide how much to invest in product variety and in customer satisfaction (e.g., via better product functionality or upgrades). Several interesting questions then arise: Are investments in the product portfolio size and in customer satisfaction complementary to each other or are they substitutes? Does the relationship between the two variables depend on the profitability of a market? How does the competitive environment drive both investment decisions?

This paper aims to answer these questions. First, we provide a game-theoretical model that studies the interplay between product variety and consumer satisfaction. We derive novel predictions regarding the relationship between the two variables and also how the profitability of a market segment as well as the number of competitors affect product variety. Second, we test our predictions using data from the video game and mobile application industry and show that our empirical evidence is supportive of the hypotheses derived from the theoretical analysis.

Our theoretical model considers an oligopoly in which firms may sell multiple products. Each firm chooses its customer satisfaction level and the number as well as the prices of products it offers to consumers. Broader product variety and higher customer satisfaction levels are costly but generate higher demand. However, the two variables target different consumer groups. An investment in customer satisfaction helps retaining those consumers who already bought a product from the firm, as these consumers are then less likely to switch to an alternative. By contrast, a broader product variety (and a lower price) increases demand from consumers who are not satisfied with their previous purchase and are therefore willing to switch. By supplying a broader product portfolio, a firm is more likely to offer a match with a consumer's preference and therefore attracts more new customers.

As product portfolio size and customer satisfaction attract different consumer groups, they are substitutes for an individual firm. However, between firms, these two instruments can either be strategic substitutes or strategic complements. In particular, a competitor's choice to invest more in customer satisfaction implies that consumers are more likely to be satisfied with the product they bought. Therefore, they are less eager to try out a new one, and investment in product variety becomes *less* profitable. By contrast, if a competitor increases its product variety, leaving a consumer unsatisfied is more costly to the firm as this consumer is likely to switch and find an alternative match with a product from the rival. Therefore, the optimal reaction of a firm is to invest *more* in customer satisfaction.

Second, we determine how the value of a market segment shapes product variety. We find an inverted u-shape relationship: if the market value is small, optimal product variety increases in the segment's profitability, whereas for high value levels, the relationship is negative. The former result is as expected, as investment in product variety is more profitable the larger the market value. Instead, the latter result is perhaps more surprising. Its intuition is rooted in the effect that the relation between product variety and customer satisfaction can be negative. In high-value segments, it is particularly profitable for each firm to retain its previous consumers by providing a high satisfaction level. This leads to few switching consumers, implying that firms optimally offer a more narrow product variety.

Third, we analyze how the number of rival firms affects optimal product variety, and show that the number of products rises with competition. The intuition for this result is again based on the interaction between the firms' two strategic decisions. With many firms in the market, each product has only a small market share, which implies that investment in customer satisfaction is less profitable. This leads to a larger number of switching consumers, which renders investment in product variety more profitable to attract these consumers.

The predictions of our model differ from those that would be obtained in classic models of imperfect competition where the focus is on product variety. For example, allowing for multi-product firms in a location model (e.g., Salop, 1979) or a representative-consumer framework (e.g., Singh and Vives, 1984) would lead to very different results: product portfolio size is larger in more valuable markets and decreases in the number of competitors. We show that these results can revert when taking into account that product variety interacts with customer satisfaction.

To test the predictions of our theoretical model, we need markets which come with both sufficient variation in the relevant variables—i.e., the number of products, consumer satisfaction, segment value, as well as competition—and with accurate measures or at least proxies of these relevant dimensions. Digital markets often fulfill these two main requirements. Their advent has led to increased product variety through lowering costs and gave consumers the ability to voice their post-consumption satisfaction through ratings (Goldfarb and Tucker, 2019). Digitized products often have relatively small marginal costs in highly dynamic and innovative markets with short life cycles, which requires firms to invest.

For our empirical analyses, we choose to study the markets for video games and apps. Both markets not only play an important role in many consumers' daily routine, they are also economically highly relevant with global turnovers of 222 and 503 billion USD in 2022, respectively.[5] While they share the features we need for our analysis, the two markets also differ in important ways. Most importantly, apps generally do not rely on prices for monetization and require lower sunk cost of production compared to video games. Having

[5]See `https://www.idc.com/getdoc.jsp?containerId=prUS49166522` and `https://www.data.ai/en/go/state-of-mobile-2023/`.

data on both markets allows us to capture the model in its entirety and provides robustness to the theoretical results.

The data for video games is directly retrieved from Steam, the leading platform for the online distribution of video games, and comprises comprehensive information on the universe of about 40,000 games available in August 2022. For apps, we use web-scraped data from Kesler (2022) on about half a million apps on Google's Play Store from January 2021 that encompasses a rich set of app and developer characteristics.

We measure product variety based on the number of titles released by developers within a year in each segment as defined by the platforms, while we infer consumer satisfaction from user ratings. Segment value is approximated by the purchase price for video games and by the number of installations for apps. We consider the latter as a proxy for attracting demand as prices play a minor role with many apps being for free. Finally, we measure competition as the number of other developers in the focal markets.

In order to account for endogeneity from linking product variety with the other core variables satisfaction, segment value, and competition, we employ two types of instruments. First, we exploit policy shocks, both by the platforms and regulatory authorities, where policies of the platform affect the design or importance of the rating system, while regulations affect the level of entry as well as exit on our markets. Second, we retrieve secondary data for both markets to infer the prevalence of pre-programmed code in the segments functioning as cost shifters and use these as instruments for value.

We find a negative relationship between the measures of product variety and consumer satisfaction for both the video game and mobile application industry; hence, in these industries, the two variables are (strategic) substitutes. For both markets, we find an inverted u-shape for the link between segment value and product variety, while competition is positively related with portfolio size.

Overall, we find strong empirical support for the hypotheses from the theoretical model. As these hypotheses contradict the conventional wisdom regarding product-line choice, they underscore the relevance of the novel effects we elicit in our theoretical model.

Related literature:

Our paper contributes to the literature on product variety choice and customer satisfaction. We discuss each literature strand in turn.

The theoretical literature on product variety choice focuses primarily on how competing firms' product lines differ. Brander and Eaton (1984) consider competition between horizontally differentiated duopolists and determine under which conditions each firm prefers to offer close substitutes or more distant products. Klemperer (1992) as well as Klemperer and Padilla (1997) analyze the effect of shopping costs on product line length and on horizontal differentiation between firms' products.[6] Studying vertical product differentiation, Champsaur and Rochet (1989) find that firms will never choose overlapping product lines, and Johnson and Myatt (2003) develop a model of quality-differentiated products and show how product lines change with entry.[7] None of these papers considers the interplay between investment in customer satisfaction and product variety, which is the focus of our study. Specifically, we develop a novel model that allows us to answer questions on the effects of this interplay for firms' strategies.[8]

In the empirical literature, several studies analyzed the profitability of investing in product variety. In this vein, Sorenson (2000) using data from computer workstation manufacturers, finds that product variety is more valuable in uncertain markets and when firms offer only few products. Eggers (2012) as well as Barroso and Giarratana (2013) analyze breadth versus depth of product lines and determine how experience and product complexity affect the relation between product-line length and firm performance.[9] Cottrell and Nault (2004) show that multi-product firms in the microcomputer software industry often benefit from economies of scope in consumption, as consumers prefer to buy products from the same firm. Hui (2004) analyzes the severeness of self cannibalization arising from larger product lines, and Nerkar and Roberts (2004) as well as Bayus and Agarwal (2007) study the effects of the time period a firm is active in the market on product line extensions.[10] We contribute to this literature by studying how optimal product variety is driven by the interaction with customer satisfaction, thereby testing hypotheses that are derived in our theoretical model.

The literature on customer satisfaction is mainly empirical.[11] The papers in this literature provide overwhelming evidence that larger customer satisfaction improves firm performance because it increases consumers' loyalty to a firm and therefore the repurchase intent and decision.[12] For instance, Anderson and Sullivan (1993) develop a utility-oriented framework to study the main drivers behind this effect. Mittal and Kamakura (2001) as well as Szymanski

[6]Carlton and Dana (2008) show that demand uncertainty induces a firm to offer a larger product line in case of considerable sunk costs, as this reduces expected costs from excess capacity.

[7]Lancaster (1990) provides a survey, discussing in detail how a firm's optimal product variety depends on demand conditions.

[8]A different strand is the literature on proliferation strategies as entry deterrents. In particular, Schmalensee (1978) demonstrates how an incumbent firm can strategically introduce new products to deter entry. This idea has been generalized and extended by e.g. Judd (1985), Bonanno (1987), Shaked and Sutton (1990), and Gilbert and Matutes (1993). For an empirical analysis, see Bayus and Putsis (1999).

[9]Kekre and Srinivasan (1990) find that the benefits of a larger product variety tend to outweigh the associated costs in many manufacturing industries.

[10]Berger et al. (2007) use laboratory experiments to precisely determine why multi-product firms obtain higher demand; they find that consumers perceive these firms to have better expertise or core competency.

[11]Hauser et al. (1994) provide a theoretical model to study the trade-off between long-term goals of firms to invest in customer satisfaction versus more short-run incentives of employees.

[12]Customer satisfaction is defined, in a broad sense, as a consumer's post-consumption judgment of whether a product provided a pleasurable level of usage-related fulfillment (see e.g. Oliver, 2014).

and Henard (2001) study how consumer characteristics moderate this effect. Anderson et al. (1997) find that a trade-off between productivity and customer satisfaction exists and determine whether it differs between goods and services. Otto et al. (2020) and Hult et al. (2022) provide recent comprehensive surveys of this literature. In contrast to these papers, we study how consumer satisfaction and product variety relate to each other. To this end, we use the finding from the literature that customer satisfaction affects the repurchasing decision in our theoretical model, and analyze the resulting implications on product variety theoretically and empirically.

The paper unfolds as follows: Section 2 sets out the model. Section 3 solves for the equilibrium, presents the results, and states the hypotheses predicted by the model. Section 4 describes the data, and Section 5 lays out the identification strategy. Section 6 then provides the empirical results. Finally, Section 7 concludes.

2 The Model

We set up a model that allows us to study the interaction between investment in product breadth and investment in customer satisfaction. Both investments are costly for firms but help them to raise demand. The former investment increases demand from consumers who look for a new product, whereas the latter investment aims to retain current consumers. To capture these pertinent features of several markets, we consider the following model.

Firms. There are M firms, denoted by $i = 1, ..., M$. At the outset (e.g., at time $t = 0$), each firm i sells m_{i0} products. Firms compete (at $t = 1$) in three strategic variables: first, each firms chooses the number of products (i.e., its product breadth); second, it sets a price for each of its products; third, it invests in customer satisfaction to retain its consumers.

We denote the number of products of firm i by m_i and the price of product $\ell = 1, ..., m_i$, by $p_{\ell,i}$. For simplicity and to be able to apply differentiation techniques, we treat m_i as a continuous variable.[13] The costs of offering a product portfolio of size m_i is $f(m_i)$, with $f'(m_i) > 0$. To ensure interior solutions, we assume that $f(m_i)$ is weakly convex (i.e., $f''(m_i) \geq 0$).

In addition to attracting consumers through a large product variety and low prices, a firm can increase its demand by investments which help to retain consumers who are currently buying its products. This can be done by, for instance, offering products with better functionality or by providing product updates. We refer to such investments as investment in customer satisfaction and denote that of firm i by s_i, $i = 1, ..., M$. A higher s_i

[13] This is in line with most papers on multi-product firms, such as Dewan et al. (2003), Johnson and Myatt (2003), or Hamilton (2009).

induces current consumers of firm i to repurchase one of the firm's products with a higher probability. The cost of investment into customer satisfaction is $c(s_i)$, with $c'(s_i) > 0$ and $c''(s_i) > 0$. We assume that s_i applies to all of firm i's products, that is, a firm incurs the associated investment cost only once (instead of on a per-product basis). This assumption is motivated by the fact a firm usually benefits from economies of scope for investment in customer satisfaction, as e.g. an improvement in the software or the color quality can be applied to multiple products. The assumption is, however, not crucial for the results.[14]

We consider a simultaneous game between firms, that is, firms choose the variables product variety m_i, product prices $p_{\ell,i}$, and customer satisfaction s_i at the same time. Therefore, our solution concept is Nash equilibrium. The assumption of simultaneous choices simplifies the presentation of the analysis and is reasonable in digital markets where products can be easily added or withdrawn, and changes in the functionality of products (e.g., via adaption of the software code) can occur in a fast way, thereby making these choices not necessarily more long-term compared to price choices. However, our main effects carry over to a situation with sequential choices in which s_i and m_i are chosen before prices are set.[15]

Consumers. There is a mass 1 of consumers. Each consumer has a valuation of v for the product. At the outset, each consumer buys one of the products from the firms. To simplify the exposition, we assume that each product existing at $t = 0$ is bought by the same number of consumers, which implies that the mass of consumers for each product at $t = 0$ is

$$\frac{1}{\sum\limits_{j=1}^{M} m_{j0}}. \tag{1}$$

If a firm invests more in customer satisfaction, a consumer who has previously bought a good from the firm, will buy from the firm again with a higher probability. To express this in a simple form, we assume that a consumer of firm i sticks to a product of the firm with probability s_i and buys an alternative product with probability $1 - s_i$.[16] This formulation is consistent with papers modeling the choice of variety-seeking consumers (e.g., Givon, 1984, or Zeithammer and Thomadsen, 2013). If a consumer is satisfied with a product, she buys again with a higher probability, but she may nevertheless occasionally like to try an alternative.[17]

[14] In particular, all qualitative results are unchanged if we instead consider product-specific investments in satisfaction (i.e., no economies of scope). We present this analysis in Appendix B.1.

[15] We show this in Appendix B.2.

[16] At the end of this section, we state an assumption on $c(s_i)$ that guarantees that the investment in customer satisfaction does not exceed 1 in equilibrium, which must hold as s_i is a probability.

[17] In Appendix B.3, we present an extension of the model in which an increase in s_i not only helps to retain existing consumers but also to attract switching consumers, and show that our qualitative results are

If a consumer decides to buy an alternative product, she considers all products available in the market. Firms offer different products, and consumers have heterogeneous tastes for these products. The demand for a product is the larger, the higher the net utility—i.e., valuation minus price—offered by the product. Formally, the probability that a switching consumer buys product ℓ of firm i (with $\ell = 1, ..., m_i$ and $i = 1, ..., M$) is

$$\frac{(v - p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{j=1}^{M} \sum_{\ell=1}^{m_j} (v - p_{\ell,j})^{\frac{1}{\beta}}}, \tag{2}$$

with $\beta > 0$.[18] The parameter β can be interpreted as the extent of consumer heterogeneity. If β is close to zero, all switching consumers buy the same product with a very high probability: since the gross utility is the same for all products and equal to v, almost all switching consumers will buy the product with the lowest price. This is equivalent to the case in which consumers are relatively homogeneous in their preferences, which leads to intense price competition. Instead, if β tends to infinity, the distribution of switching consumers across products becomes almost uniform, regardless of the prices charged. This represents the case in which consumers are highly heterogeneous in their tastes: different consumers then buy different products and decide only according to their preferences, while prices only play a secondary role. For intermediate values of β, the model can be interpreted as one in which consumers decide partly based on the price and partly according to their preferences. Another interpretation of the formula in (2) is that there are search frictions by consumers when choosing which products to buy—e.g., they are not fully informed about all available products. These search frictions are more pronounced if β is larger. To guarantee interior solutions, we assume that $\beta > (M - 1)/M$, that is, consumers exhibit some degree of heterogeneity. Otherwise, second-order conditions with respect to product prices may not be satisfied.[19]

Our model of consumer demand is a variant of a standard urn-ball matching function (see e.g., Petrongolo and Pissarides, 2001), where products take the role of balls. However, in contrast to the classic urn-ball matching function, the balls have different matching probabilities (i.e., probabilities to attract consumers) and firms can influence the matching probability with the prices they set for their products.[20] Therefore, firms can attract

still valid then.

[18] As consumers obtain a gross utility of v from each product, the net utility of product ℓ of firm i is $v - p_{\ell,i}$.

[19] The formulation in (2) implies that a switching customer who bought from firm i in the past will buy from firm i again if she finds one of firm i's products most attractive. This assumption is made to ease the exposition.

[20] See Fainmesser and Galeotti (2021) for a similar formulation in a different context (i.e., influencers in social media choose the quality a follower obtains, and an influencer probabilistically gains more followers

switching consumers by a larger product variety and lower prices.

To simplify the exposition, we focus on a situation with symmetric firms, that is, all firms offer the same product portfolio size at the outset: $m_{i0} = m_{j0}$ for all $i, j = 1, ..., M$. We denote this size by m_0; hence, (1) is given by $1/(Mm_0)$. This assumption is not crucial for the results.[21]

Finally, to ensure that solutions are interior, that is, the profit function of a firm is quasi-concave and $s^* \leq 1$ as it is a probability, we assume that the cost function for investment in customer satisfaction is sufficiently convex. Specifically, we assume $c''(\cdot) > v(M-1)^2/(\beta M^3)$ and that the third derivative of $c(\cdot)$ is either negative or, if it is positive, then small relative to the second derivative.

3 Analysis and Results

In this section, we solve for the Nash equilibrium of the game. We analyze the interplay between investments in product variety and customer satisfaction and determine how the product variety chosen by firms in equilibrium changes with the profit per consumer and the number of competing firms.

Denoting by $\mathbf{m} = \{m_1, ..., m_M\}$ and $\mathbf{s} = \{s_1, ..., s_M\}$ the vectors of product varieties and investments in customer satisfaction levels, respectively, and by $\mathbf{p} = \{p_{1,1}, ..., p_{m_1,1}, ...p_{1,M}, ..., p_{m_M,M}\}$ the vector of firms' product prices, the profit function of firm i is[22]

$$\Pi_i(\mathbf{m}, \mathbf{s}, \mathbf{p}) = \frac{s_i}{M}\left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{\ell=1}^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}}\right) + \frac{\sum_{j=1}^{M}(1-s_j)}{M}\left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{j=1}^{M}\sum_{\ell=1}^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}}\right) - f(m_i) - c(s_i).$$

The first term represents the revenue from satisfied consumers of firm i. Since firms are symmetric at the outset, a consumer mass of $1/M$ buys one of firm i's products. These consumers will buy a product of firm i again with probability s_i. Each retained consumer chooses one of the m_i products offered by firm i, and is more likely to buy the one that gives the highest net utility. The second term is the revenue from unsatisfied consumers of the firms. A fraction $1 - s_j$ of consumers of each of firm j potentially switches. The probability that such a consumer chooses product ℓ of firm i is given by (2), and firm i offers

when offering higher quality).

[21]In Appendix B.4, we show that our findings carry over to the case with asymmetric firms.

[22]We write the number products of each firm as a discrete variable rather than a continuous one because the different terms are then easier to understand. When solving for the equilibrium, we treat the variable as continuous and apply differentiation techniques.

m_i products. Finally, the third and the fourth term are the costs of investing into product variety and customer satisfaction, respectively.

Taking the derivatives of $\Pi_i(\mathbf{m}, \mathbf{s}, \mathbf{p})$ with respect to m_i, s_i, and $p_{\ell,i}$, $\ell = 1, ..., m_i$, we obtain the first-order conditions.[23] As a firm optimally sets the same price for each of its products, the first-order conditions can be written as

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{s}, \mathbf{p})}{\partial m_i} = \frac{\sum_{j=1}^{M}(1-s_j)p_i\,(v-p_i)^{\frac{1}{\beta}}\left(\sum_{j=1,\, i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0, \tag{3}$$

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{s}, \mathbf{p})}{\partial s_i} = \frac{p_i}{M}\left(1 - \frac{m_i(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right) - c'(s_i) = 0, \tag{4}$$

and

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{s}, \mathbf{p})}{\partial p_{\ell,i}} = \frac{s_i}{M m_i} + \frac{\sum_{j=1}^{M}(1-s_j)\,(v-p_i)^{\frac{1-\beta}{\beta}}}{M\beta\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}\left(\frac{p_i\,(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}} - (p_i - \beta(v-p_i))\right) = 0. \tag{5}$$

The first-order conditions with respect to m_i and s_i—i.e., (3) and (4)—show the trade-off a firm faces when increasing either of the two variables: both an increase in m_i and an increase in s_i expand firm i's demand (first term in (3) and (4), respectively), but the increase is costly for the firm (second term in (3) and (4), respectively). An important difference between the two strategic variables is that they work in different ways. In particular, investment in customer satisfaction helps a firm to keep more of those consumers who are presently buying one of firm i's products. Instead, raising product variety increases the demand from switching consumers, regardless of whether they bought a product from firm i or from its competitors. Specifically, the first term in (4) is a multiple of firm i's consumers at the outset, $1/M$, whereas the first term in (3) is a multiple of the average mass of all firm's switching consumers, $\sum_{j=1}^{M}(1-s_j)/M$.

Turning to (5), when increasing the price $p_{\ell,i}$, $\ell = 1, ..., m_i$, of one of its products, firm i obtains a larger profit from its satisfied consumers, which is expressed in the first term of

[23] Given our assumptions that $f''(m_i)$ is larger than zero and $c(s_i)$ is sufficiently convex, the second-order conditions for profit maximization are satisfied.

(5). Instead, for switching consumers, the firm faces the standard trade-off that a higher price induces fewer consumers to buy from the firm, but the firm obtains a larger profit from those consumers who still find one of the firm's products most attractive. This trade-off can be seen in the second term of (5), which contains positive and negative elements.

It is instructive to first determine the interplay between product variety choice and investment in customer satisfaction, both for a single firm and between firms. This leads to the following result:[24]

Result 1. *(i) Holding all other variables constant, the relation between investment in product variety and investment in customer satisfaction of firm i is negative.*
(ii) Holding all other variables constant, investment in product variety of firm i and investment in customer satisfaction of firm j can either be strategic substitutes or strategic complements.

The intuition behind part (i) of the result is that investment in product variety and investment in customer satisfaction target different consumer groups.[25] While investment in customer satisfaction aims at retaining current consumers of a firm, a larger product variety aims at attracting more switching consumers. If a firm invests more in customer retention, more consumers are satisfied and are less likely to switch. Therefore, a larger product variety, which is intended to obtain demand from switching consumers, is *less* profitable. Similarly, an increase in a firm's product-line length helps to gain more switching consumers. As a consequence, investment in consumer satisfaction is *less* profitable. Therefore, the relation between these two variables is negative.

Turning to part (ii) of Result 1, we obtain a similar result as in part (i) when determining how firm i reacts with its product variety to a change in the customer satisfaction level of a rival firm j. A higher customer satisfaction level of firm j induces firm i to reduce its product variety, which implies $dm_i/ds_j < 0$. The intuition is again that a larger investment in customer satisfaction by firm j leads to a smaller mass of switching consumers. As the reason for increasing product variety is to attract switching consumers, firm i benefits *less* from increasing its product variety.

However, determining firm i's response in customer satisfaction investment if firm j offers an increased product variety, yields $ds_i/dm_j > 0$. Therefore, a larger product variety of firm j induces firm i to invest more in customer satisfaction. The intuition is that a larger product variety of a rival implies that the rival receives a higher demand from switching consumers. Therefore, firm i loses more switching consumers to its rivals. Consequently,

[24]The proofs of this and all other results can be found in Appendix A.

[25]We note that the result determines the relation between product variety and customer satisfaction, holding prices fixed. It holds for any constant prices and not just e.g. equilibrium prices.

the firm has a stronger incentive to retain its consumers and will therefore invest *more* in customer satisfaction.

Due to the different signs of dm_i/ds_j and ds_i/dm_j, it is not clear whether the two variables are strategic substitutes or complements. They are strategic substitutes if the effect arising from $dm_i/ds_j < 0$ dominates, but strategic complements if $ds_i/dm_j > 0$ dominates.

In summary, holding all other variables fixed, the relation between investment in product variety and customer satisfaction is not clear. There is, however, a strong indication that the relationship between the two variables is negative because for an individual firm, the two variables are substitutes. In our empirical analysis, we will determine the relation between the two variables.

Result 1 provided insights into the functioning of the model and the interaction between product variety and customer satisfaction, holding all other variables fixed. We now turn to the question of how *in equilibrium*—i.e., in which all decisions of firms are taken into account—product variety is affected by the market environment. In the unique symmetric equilibrium of the game, that is, $m_1^* = \cdots = m_M^*$, $s_1^* = \cdots = s_M^*$, and $p_{1,1}^* = \cdots p_{m_M,M}^*$, we denote the equilibrium product portfolio size by m^*, the equilibrium customer satisfaction level by s^*, and the equilibrium product prices by p^*. These equilibrium values are implicitly characterized by the first-order conditions, which, due to symmetry, can be simplified to[26]

$$\frac{(1-s^*)(M-1)p^*}{m^*M^2} - f'(m^*) = 0, \quad \frac{(M-1)p^*}{M^2} - c'(s^*) = 0, \tag{6}$$

and

$$\frac{M\beta\,(v-p^*) - (M-1)p^*\,(1-s^*)}{m^*M^2\beta\,(v-p^*)} = 0. \tag{7}$$

Using these equations, we can now perform comparative-static analyses to determine how the equilibrium product variety $m^\star$ changes with the market environment. We first consider consumer valuation v:

Result 2. *The relation between the per-consumer value v and the equilibrium number of products is non-monotonic—i.e., $m^\star$ is increasing in v for low values of v and decreasing for high values of v.*

The result shows that a more valuable market (i.e., a larger v) may induce firms to offer a lower variety of products.[27] This is potentially counter-intuitive: if a market segment is

[26]In Appendix B.5, we provide an example with concrete cost functions that allows for a closed-form solution of the model.

[27]We note that v can also be interpreted as the difference between the consumers' valuations and the firms' production costs. In this respect, an increase in v can either result from consumers valuing products more or from a fall in production cost, as both effects make the market more valuable.

more valuable, firms usually find it more profitable to offer a larger number of products. In particular, if a market is more valuable, the equilibrium price and therefore also the equilibrium profit per product is larger, as p^* is increasing in v. Since the cost of introducing an additional product is not affected by v, this implies that—*ceteris paribus*—firms should optimally expand their product range.

However, in our model there is a strategic countervailing force. In a valuable market segment, each firm has a strong incentive to retain its consumers. It will therefore invest more in customer satisfaction. This implies that fewer consumers switch away from the firms, which also helps the firm to charge higher prices. Since all firms invest more, the number of switching consumers falls, which implies that each firm can attract only fewer consumers with its product variety. As a consequence, firms respond with a reduction in the number of products they offer. The result therefore occurs because of the interplay between investments in customer satisfaction and in product variety. In fact, if s_i were fixed for all firms $i = 1, ..., M$, an increase in v would unambiguously lead to an increase in the number of products. However, this result no longer holds if customer satisfaction is chosen endogenously.

Result 2 shows that the countervailing effect dominates if v is sufficiently large. The intuition is as follows: If v is large, firms optimally invest a lot in customer satisfaction to retain many of its consumers. Consequently, investing in an additional product does not pay off much. By contrast, if v increases starting from a low level, the effect that the market segment becomes more valuable dominates; hence, each firm optimally increases both its product portfolio size and its investment in consumer satisfaction.

Second, we turn to the number of firms M:

Result 3. *The relation between the number of firms and the equilibrium number of products of each firm is positive—i.e., $m^\star$ is increasing in M.*

Result 3 shows that an increase in competition caused by a larger number of firms induces firms to expand their portfolio size. To understand the intuition behind the result, it is helpful to distinguish between the situations in which the number of firms is relatively small and in which the number of firm is relatively high. If there are only few firms, a large number of switching consumers will buy from each firm, even if its product variety is only small. If competition increases, a firm can counter the effect that fewer switching consumers buy from it by increasing its product variety. Although more competition leads to a fall in prices, the increase in product variety allows a firm to dampen this price reduction. Therefore, for a small number of competitors, the result that product variety increases in competition results from the fact that a firm would lose a considerable number of switching consumers when offering only few products. It is therefore not driven by the interplay between the investments in product variety and in customer satisfaction.

However, if the number of firms is relatively large, the result depends on this interplay. An increase in the number of firms implies that each firm has a lower market share of its existing products. Consequently, investing in customer satisfaction pays off less. If each firm invests less in customer satisfaction, more consumers switch, which also leads to a fall in prices. Offering a larger product variety is then profitable as there is potentially more demand and also helps to stabilize prices.

Before presenting the empirical predictions arising from our results, we note that in our model firms can charge positive prices and therefore maximize over these prices. However, many digital markets are characterized by zero or negligible prices. Instead of maximizing profits, the incentive of firms is to maximize demand, reflected e.g. for developers of apps by downloads or installations. In Appendix B.6, we show that all our Results 1-3 are *still valid* in that case.

Empirical Predictions

We now briefly state the empirical predictions that arise from our theoretical analysis, which we test in the next sections using data from video games and apps. From Result 1, a central effect in our model is that investment in customer satisfaction and investment in product variety can either be (strategic) substitutes or complements, which can be translated into the following hypothesis:

Hypothesis 1. *The relation between the level of customer satisfaction and a firm's number of products can be positive or negative.*

As stated after Result 1, our model provides a clear intuition why the relation between the two variables is more likely to be negative. However, we let the data show which effect will be dominating here.

Results 2 and 3 determine how product variety changes with the competitive conditions in the market. Result 2 states that the value of a market segment affects the equilibrium product portfolio size in a non-monotonic way and can be translated into the following hypothesis:

Hypothesis 2. *The effect of a higher profit per consumer on a firm's number of products follows an inverted u-shape.*

Finally, Result 3 relates the equilibrium product portfolio size to the number of firms:

Hypothesis 3. *The effect of a larger number of firms on a firm's number of products is positive.*

Difference to Standard Models

To conclude this section, we point out that the results derived from our model are genuinely due to the interaction between customer satisfaction and product variety. In particular, the results differ from those of classic models of imperfect competition without such interaction.

Consider for example a classic model of horizontal product differentiation, such as a representative consumer model (e.g., Bowley, 1924; or Singh and Vives, 1984) or a location model (e.g., Salop, 1979), with a constant number of firms. In both types of models, if consumer valuations increase, equilibrium prices will (weakly) rise, implying that investment in product variety becomes more profitable as well. Therefore, the relation between profit per consumer and the number of products is positive—contrary to our Result 2.

If there is free entry by firms and the market becomes more valuable, more firms enter (e.g., Mankiw and Whinston, 1986). In these types of models, firm entry leads to a reduction of each firm's product variety. This implies that, under some parameter constellations, a more valuable market induces firms to offer a lower product variety, which is in line with our Result 2.[28] However, this effect implies a negative relationship between equilibrium product variety and the number of firms, which is in contrast to our Result 3.

Therefore, obtaining Results 2 and 3 jointly in one model is not possible in these classic frameworks. In fact, the aim of these frameworks is to model price competition of firms that (in some cases) can also invest in product variety. However, they are not concerned with consumers who have already bought the products of the different firms, which implies that concerns about consumer satisfaction and retention strategies are not present. However, such concerns are important in many markets, where consumers buy products repeatedly. The novelty of our model is to consider this interaction: the model allows for joint investment decisions in customer satisfaction and product portfolio size, which leads to novel insights.

4 Data

We test the theoretical predictions using data from the video game and mobile application industry. In this section, we describe the data sets and explain how the variables are mapped from the theory to the empirics along with a provision of descriptive statistics.

[28]Details of the analysis are available from the authors upon request.

4.1 Data sets

4.1.1 Games

To assemble our data set on games, we scraped Steam, the leading platform for online distribution of video games, in August 2022. We collected information on all games listed on the Steam store[29] at that time and retrieved the corresponding information from Swiss IP addresses.[30] This generated a data set, which we in turn validated by consulting several external sources (SteamDB, SteamSpy, VG Insights) that reported similar amounts. This gives us reassurance that the sample indeed constitutes the universe of important games. We drop video games with missing information on our core variables publisher identity, price, ratings, and release date.[31] In addition, we exclude games with implausible release dates like dates which lie in the future or which are missing entirely. This leaves us with 38,939 video games.

4.1.2 Apps

Our app market data are those used by Kesler (2022), which comprises apps from the Google Play Store in January 2021.[32] We chose January 2021 because later that year Google changed the display of user ratings, one of our core explanatory variables, which led many developers to no longer provide this information.[33] In order to be sure that the data includes all important apps (i.e., those with at least 10,000 installations), we turn to figures provided by Androidrank, a platform that collects and provides data on the Google Play Store. We find a coverage of 90 percent of our data compared to the Androidrank data. The majority of the sample, however, has fewer installations than 10,000, thereby constituting the characteristic long tail. As for the games market, we exclude apps with missing information on our key variables. This leaves us with a total of 505,449 apps.

[29] See `https://store.steampowered.com/search/?sort_by=Price_ASC&category1=998`.

[30] Having data from one period in time may lead to missing firms (and products) from the past that exited until that date. We address this so-called survivorship bias in Appendix C.1 by showing that the vast majority of video games remains in the market over time, as the costs to stay active are low.

[31] See Appendix C.1 for details on the distinction between developers and publishers along with our definition of a firm for both markets.

[32] The Google Play Store also comprises games in addition to apps, where apps make around 80% of offers. We focus on apps in our analysis to have a second market that is sufficiently different from video games.

[33] See `https://android-developers.googleblog.com/2021/08/making-ratings-and-reviews-better-for.html`.

4.2 Variables measurement

Our two data sets contain either direct measures of the core variables of our model, portfolio size, satisfaction, segment value, and competition, or empirical proxies thereof.

Product variety as our first core variable is directly measured by the number of products in a market segment in a given release year. We restrict the measure to one year to better reflect the short-lived product cycles in the digital sphere, as in Ozalp and Kretschmer (2019).

For games, we define segments following Rietveld and Ploog (2021) who use the Steam classification to distinguish between the ten different game genres: action, adventure, casual, massively multi-player, racing, RPG, simulation, sports, strategy, and casual. This maps into 293 segment-years. For apps, we use the definition of segments provided by Google's Play Store which distinguishes 31 separate categories.[34] This leads to 378 different segment-years in the app market. In the following, we use the term "segment" for segment-years to ease readability.

The second core variable of our theoretical analysis is satisfaction, which is proxied by the share of positive user reviews on Steam (games) and the average rating that users provided on Google's Play Store (apps). In our robustness checks, in Section 6.2, we show that consumer ratings are highly correlated with updating frequency, as a proxy of quality, and ratings of professionals, which is a potentially more objective measure of satisfaction.

The third core variable is the value of a market segment—e.g., consumers' willingness-to-pay—which we do not directly observe for either games or apps. For games, we use the price of the game as our value proxy, since prices should come close to consumer valuations, deeming both variables to be highly correlated. Indeed, as mentioned above, in our theoretical model, equilibrium prices increase in the consumers' valuation v. While some apps indeed have prices—be it prices for the purchase of an app or prices for in-app purchases—the low prevalence implies that they may not be good proxies for consumer valuations. A better measure for consumer value is the demand for an app, which we proxy by the number of installations. The measure is reported in twenty size categories of which we conservatively take the lower bound. The idea here is that installing an app means that the utility a consumer derives from the app is larger than the nuisance of installing it. The number of installations hence constitutes a lower bound on segment value and more installations of an app imply higher utility derived from the app by consumers. As explained above, in Appendix B.6 we show that all of our predictions are valid if the value of a segment is determined by

[34] The categories comprise Art & Design, Auto & Vehicles, Beauty, Books & Reference, Business, Comics, Communication, Dating, Education, Entertainment, Events, Finance, Food & Drink, Health & Fitness, House & Home, Libraries & Demo, Lifestyle, Maps & Navigation, Medical, Music & Audio, News & Magazines, Parenting, Personalization, Photography, Productivity, Shopping, Social, Tools, Travel & Local, Video Players & Editors, and Weather.

consumer demand, and a firm's (developer's) objective is to maximize demand (number of installations).

Our last core variable from the theoretical model is competition, which we measure by the number of competing producers in a focal segment.

Table 1 provides an overview of how we empirically measure or proxy the core variables of our theoretical model.

Table 1: **Mapping theory to empirics**

		Empirics	
	Construct	Games	Apps
Theory	Variety	# Own Products	# Own Products
	Satisfaction	% Positive Reviews	Average Rating
	Value	Price	# Installations
	Competition	# Other Publishers	# Other Publishers

Importantly, our analysis is at the segment-level. Our dependent variable is the number of products of the own publisher in the respective segment. The explanatory variables are aggregated to the segment-level by taking averages, implying that dummy variables are translated into shares. In the regression analyses, we additionally have the month of release and the year of release as well as segment dummies as control variables (see Rietveld and Ploog, 2021).

Given the skewness of our core variables and the many zeroes in the number of competitors, we apply the inverse hyperbolic sine (IHS) transformation. We opt for the IHS transformation instead of the still more popular logarithmization since it does not imply adding an arbitrary positive number to the zeroes in the data while still retaining the interpretation of the log transformation (Bellemare and Wichman, 2020).

4.3 Descriptive statistics

Table 2 displays descriptive statistics for our four core variables. Portfolio size is low on average for both games and apps where both medians are 1. There is, however, substantial variation with the maximum number of products by the publisher in the respective segment-year being 92 for games and 965 for apps. Regarding satisfaction, 46% of the consumers leave a positive review on Steam and the average rating is 4.04 for Play Store apps on a scale from 0 to 5. There is also substantial variation in our two proxies for segment value. For games, mean segment-specific prices range between 0 and 17CHF ($\approx$ 17USD) with both a mean and a median of around 7.7CHF. Our proxy of segment value for apps is the number of installations which vary between 1,000 and 28 million. The average number of installations is 163,108 with a median of less than a half of this figure. Competition is intense for both games and apps compared to traditional markets with a mean number of products by competing publishers in the focal segment of 1,520 for games and of 4,846 for apps. Means and medians are similar in size in both cases. There are substantial differences between the minimum and the maximum number of competitors in the corresponding segments.

Table 2: **Summary statistics**

	Mean	**Median**	**SD**	**Min.**	**Max.**
Games					
Dependent variable					
# products by publisher	1.233	1.000	1.334	1	92
Key explanatory variables					
Satisfaction: share pos. ratings	0.460	0.417	0.092	0.222	0.773
Value: mean price in CHF	7.705	7.670	1.993	0.000	17.348
Comp.: mean # of other publishers in	1,520	1,355	1,007	4	3,279
# of obs.: 38,939					
Apps					
Dependent variable					
# products by publisher	1.844	1.000	6.951	1	965
Key explanatory variables					
Satisfaction: mean rating score	4.036	4.028	0.151	2	4
Value: mean # of installations	163,108	70,432	369,624	1,000	27,750K
Comp.: # of other publishers	4,846	4,030	3,564	0	13,683
# of obs.: 505,449					

5 Identification

Identification problems loom large in our empirical specification which links portfolio size to our three core explanatory variables comprising satisfaction, segment value, and competition. Factors that are important for consumer satisfaction and value of a segment (e.g., segment popularity) are usually also determining portfolio size, which leads to an endogeneity problem. Competition is likely to be endogenous via its correlation of the error term from our equation of interest: unobserved (to us) factors which drive portfolio size will be correlated with competition. In addition, segment value enters the estimation equation both in linear and quadratic form which leads to four endogenous variables in our model. Given that the squared instrumental variables for value are appropriate instruments for value squared, proper identification hence requires sets of instrumental variables for satisfaction, segment value, and competition.

We use two main sets of instruments for identification, as displayed in Table 3. These instruments constitute exogenous sources of variation, comprising (i) policy shocks for satisfaction and competition, and (ii) cost shifters for segment value. For the former two variables, we exploit policy changes introduced either by the two platforms, Steam and Google Play Store, or through legislation.

Table 3: **Overview of instruments**

Endogenous Variable	Markets		Intuition
	Games	Apps	
Satisfaction	Curation Regimes	Fake Review Removals	Policy Shocks
Value	SDKs & Engines	SDKs & Size	Cost Shifters
Competition	Delistings	GDPR	Policy Shocks

As an instrument for customer satisfaction, we use policy changes implemented by Steam and Google's Play Store, respectively. For games, we use Steam's replacement of its own curation for games to be launched on the store by community-based curation ("Steam Greenlight") on August 1, 2012, which itself was replaced by an app store-type of curation, i.e., open access ("Steam Direct") from June 1, 2017 onwards. During the Steam Greenlight regime, the launched games focussed on satisfaction because user ratings were an important aspect for the game to be released and were thus of better quality. Therefore, these policy changes are likely to be correlated with satisfaction, but unlikely to affect unobserved determinants of portfolio size. We operationalize the related instruments as the share of games which were released after and within the two policy amendments.

Unfortunately, there are no comparable policy changes at the Google Play Store. For the Google Play Store, we instead use its publicly announced removals of allegedly "fake" reviews.[35] Such a removal is directly related to our measure of satisfaction, namely user ratings, but is unlikely to be related to product portfolio size. Apps released around the respective announcements should come with a more accurate customer satisfaction and are less likely to be of low quality. We define "within" releases as apps, which were introduced within a time window of 180 days after the enactment of each declared removal and compute the share of apps per segment as an instrument.

To identify the effect of segment value on portfolio size, we use software development kits (SDKs) for both games and apps as well as engines for games (for which there is no counterpart for apps) and the installation size for apps. SDKs are standardized tools which

[35]See Appendix C.2 for a chronology of announcements that we used for our instrument.

facilitate software development of particular functionalities. A popular example of an SDK for apps is Google Analytics that allows to collect user engagement data. Engines are also pre-programmed tools but are used for more sophisticated tasks. One prominent example is the Unreal Engine by Epic Games, which is used in animation. Both SDKs and engines are tools which affect software development costs (see Miric et al., 2022) while they should be uncorrelated with other unobserved factors determining portfolio size. In a similar vein, the installation size approximates costs of programming an app and picks up possible changes over time (Boudreau, 2019).[36] We scrape information on SDKs and engines for games from SteamDB.[37] For games, we use the mean number of SDKs and the mean number of engines per segment as our main instrument for segment value. In addition, we use the interaction between both instruments and use the squared value of each instrument as an instrument for the squared segment value. While installation size is given for each app, we retrieve information on SDKs for apps from AppBrain, which infers all libraries contained in apps and distinguishes them into ones related to development, social, and advertising.[38] Consequently, we use the mean installation size, the mean number of libraries, and the mean number of development libraries along with the corresponding squared terms as our main instruments for value and squared value, calculated at the segment level.

Finally, for competition, we use Steam's delisting of games from the platform as an instrument. Such delistings can occur for various reasons, including violations of the terms of service or unethical behavior on part of the developer.[39] However, they are unlikely to be related to other unobserved factors, which determine own portfolio size while at the same time being highly correlated with within-segment competition. The data on delistings of games is hand-curated, enabled by the relatively low number of products. While there also are delistings on the Google Play Store, there is no data base which tracks delistings and the sheer size of the platform makes it impossible to hand-collect such information. We instead use the enactment of the General Data Protection Regulation (GDPR) on May 25, 2018 as an instrument for competition on the app market. Janssen et al. (2022) find that the GDPR coincided with a massive exit of low quality apps, while it, more importantly, also made entry harder through reduced revenues and higher costs. Given their much higher sunk cost of development and decreased relevance of user data for games, the enactment of the GDPR

[36]As explained in Footnote 27, in our model the segment value indeed changes with the costs of developing a product. Therefore, the instrument is also backed by our theoretical analysis.

[37]Specifically, we look at the most important technologies, i.e., those embedded in more than 100 (engines) and 1000 (SDKs) games (see `https://steamdb.info/tech/`). We retrieve the corresponding list of games to relate it back to our sample.

[38]More specifically, we take this information from Kesler et al. (2019) and crawl the AppBrain page for the remaining, mostly younger, apps.

[39]See Appendix C.2 for more details and a chronology of delistings that we used for our instrument.

had less effects on competition for games and is thus only used as an instrument for apps.

We operationalize our instrument for competition as the share of games delisted in the segment and the mean number of apps released after the enactment of the GDPR per segment.

6 Empirical results

6.1 Main estimations

Estimating an equation with four endogenous variables is a formidable task, even when plausible instruments are available. Given the complexity of our model, we resort to the "control function" (CF) method, perhaps best described in Wooldridge (2015). The CF approach essentially comes down to estimating the "first stage" equation of the set of instruments and the exogenous variables on the endogenous variables. The residuals of this first stage regression are subsequently inserted as additional regressors in the equation of interest, thereby controlling for the endogeneity bias. The control function approach and 2SLS are equivalent in many circumstances. We prefer the CF method over 2SLS (or GMM) since efficiency is a major concern for a complex model like ours and since the CF model is more efficient than 2SLS and GMM. We display GMM full information maximum likelihood estimation results in our robustness checks (see Section 6.2).[40]

Since our estimation equation contains "generated regressors" (the four residuals from the first stage), the corresponding variance-covariance matrix is no longer block-diagonal. We therefore display block-bootstrapped standard errors in our regression tables.[41]

Table 4 displays our control function OLS estimation results. It shows that our theoretical predictions are reflected by our data, both for games and for apps. First, our theoretical model predicted a negative mapping between satisfaction and portfolio size, which is what we indeed find for both games and apps. The respective coefficients are estimated with precision and indicate an approximate satisfaction elasticity for games of -0.5 and for apps of -0.3.

Second, segment value, empirically proxied by the price for games and by the number of installations for apps, is linked to portfolio size with an inverted-u shape as predicted. The coefficient estimates on both the linear and quadratic terms are separately and jointly statistically significant different from zero for both markets. Maximum portfolio size for

[40]We use the GMM full information maximum likelihood since it is the most efficient GMM estimator.

[41]Standard errors are unclustered since clustering standard errors in a bootstrap setting is computationally very intensive and because clustering at the publisher level or at the publisher-year level does not lead to qualitatively or quantitatively different estimates of our standard errors.

Table 4: **Estimation results**

	Games		Apps	
	OLS	**OLS CF**	**OLS**	**OLS CF**
	Coeff.	**Coeff.**	**Coeff.**	**Coeff.**
	(std. err.)	**(std. err.)**	**(std. err.)**	**(std. err.)**
Satisfaction	-0.222**	-0.499***	-0.301***	-0.301*
	(0.091)	(0.179)	(0.049)	(0.162)
Value	0.363**	2.157**	0.101***	0.098***
	(0.168)	(0.991)	(0.010)	(0.036)
Value^2	-0.161*	-1.023**	-0.004***	-0.004***
	(0.087)	(0.497)	(0.000)	(0.002)
Competition	0.025	0.059**	0.024***	0.074***
	(0.018)	(0.028)	(0.004)	(0.015)
1st stage residual satisfaction	—	0.357*	—	-0.042
	—	(0.194)	—	(0.172)
1st stage residual value	—	-1.896*	—	-0.003
	—	(1.087)	—	(0.038)
1st stage residual value^2	—	0.904*	—	-0.001
	—	(0.546)	—	(0.002)
1st stage residual competition	—	-0.048	—	-0.061***
	—	(0.039)	—	(0.015)
F-test for 1st stage residuals joint significance	—	1.862	—	35.710
(p-value)	—	0.114	—	0.000
Hansen-Sargan J test	—	4.344	—	17.173
(p-value)	—	0.930	—	0.103
F-test 1st stage residual for satisfaction	—	1642	—	8937
F-test 1st stage residual for value	—	1059	—	13682
F-test 1st stage residual for value^2	—	1085	—	13365
F-test 1st stage residual for competition	—	2140	—	12638

Notes: The dependent variable is the number of titles produced by the own publisher in a given segment. The specifications additionally include controls for release month, release year, and segment. The "first stage residuals" refer to the OLS residuals generated from the first stage regressions of the endogenous variables satisfaction, value, value^2, and competition on the exogenous variables and the sets of instruments. Satisfaction is measured as the share of positive ratings for games and as the average rating score on a 1-5 scale for apps. The empirical proxy for segment value in the case of games is price and it is the number of installations for apps. Competition is measured as the number of competing publishers in the own segment for both apps and games. Satisfaction, value, value^2, and competition as well as the dependent variable are all IHS-transformed. All first stage F-tests are statistically highly significant. Standard errors are bootstrapped. $* \ p < 0.10$, $** \ p < 0.05$, $*** \ p < 0.01$.

games is reached at a mean segment-specific price of 1.3CHF. Comparing this with the mean price of 7.7CHF for games implies that the majority of games are on the decreasing branch of the inverted u-shaped curve. The corresponding maximum for apps is 104,490 installations which compares to a mean of 163,108 installations, which again implies that the majority of apps are on the decreasing branch of the curve.[42]

Finally, our estimation results also provide evidence in favor of our third hypothesis that predicted a positive relationship between competition and portfolio size. The respective coefficient is estimated with precision in both markets. It are also economically significant: a one percent increase in the number of competing publishers is associated with an 0.05 percentage points increase in portfolio size for games and with an 0.07 percentage points increase in portfolio size for apps since, as for log-log transformations, IHS-IHS transformations approximately translate into elasticities.

Regarding identification, valid tests for the presence of endogeneity (given our choice of instruments) are the t-statistics of significance of the coefficients on the first stage residuals, the "CF terms". For games, they are—with the exception of the term corresponding to competition—statistically weakly significant. For apps, the only statistically significant CF term is the one related to competition. The four CF terms for the app market are, however, statistically jointly significant. Taken together, these findings suggest that there indeed is evidence for endogeneity and the need to take it into account by applying instrumental variables estimation.

For an instrument to be valid it must (i) be correlated with the endogenous variable, (ii) uncorrelated with the error term of the equation of interest, and (iii) make economic sense. We have argued in Section 5 why our set of instruments should make economic sense—i.e. fulfill property (iii). Property (i) is assessed by F tests for joint significance of the instruments in the first stage equations, which we display at the bottom of Table 4. Our F tests are all substantially larger than the rule-of-thumb value of 10 suggested by Staiger and Stock (1997) and also substantially larger than the more conservative critical value of 104.7 advocated for more recently by Lee et al. (2022). We attempt to asses property (ii) by the Sargan-Hansen test for overidentifying restrictions. The corresponding test χ^2 statistics imply that uncorrelatedness of the instruments with the error term of our equation of interest cannot be rejected at any conventional significance level.

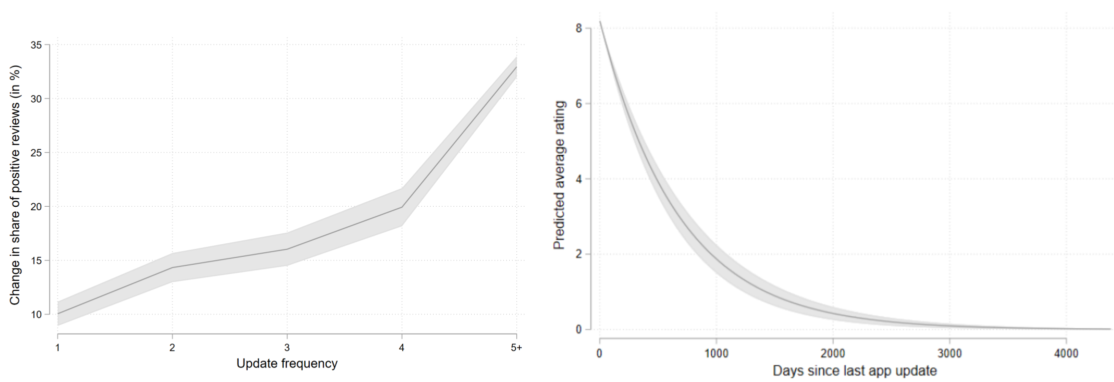
[42] To arrive at these figures, we calculate the maxima based on the IHS transformed variables and apply the hyperbolic sine transformation to these maxima.

6.2 Robustness

6.2.1 Validating the satisfaction measure

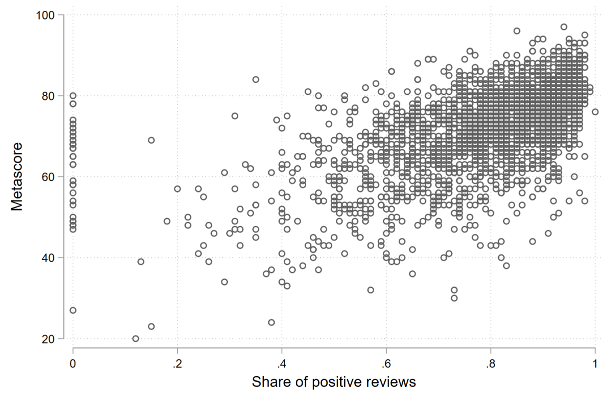
In our empirical analysis, we use consumer-reported evaluations as our proxy for satisfaction. Such evaluations might, however, be biased and are prone to manipulation which in fact is why the Google Play Store removed allegedly faked reviews, a policy intervention which we use for identification as discussed in Section 5. To verify if user evaluations are plausible proxies for actual satisfaction, we compare user ratings with two other, potentially more "objective", measures of quality: update frequency and professional ratings. More frequent updates should be correlated with higher game or app quality which should map into consumer satisfaction. We observe the actual number of updates for games, censored at five updates for illustrative purposes, and for apps we observe the days elapsed since the last update. Given the large number of observations, we cannot show simple scatter plots but need to aggregate the data instead. For games, we do so by running OLS regressions of log consumer satisfaction on four dummy variables for the number of updates for games with no updates serving as the baseline category, implying that the coefficient estimates we obtain translate into changes in consumer satisfaction conditional on the number of updates (e.g., if α denotes the coefficient estimate on one update, consumer ratings improve by $exp(\alpha) - 1$ relative to no update). We display these changes in Figure 1. For apps, we run an OLS regression of the number of days elapsed since the last update on log satisfaction and display the associated prediction in Figure 1. The shaded areas in the figures are the 95 percent confidence intervals which we calculated using the "Delta" method (Greene, 2003). The coefficient estimates on our measures of update frequency are statistically highly significant with t-values larger than 20 for both games and apps, thus validating ratings as a proxy.

Figure 1: **Update frequency and satisfaction**



Yet another way of backing up our use of consumer ratings as proxy for satisfaction is to assess whether or not consumer ratings are similar to the ratings of professional game critics since they are less likely to provide biased or even fake reviews as doing so would ruin the professional's reputation. Such data is available for games where the website `metacritic.com` provides this information for a subsample of 2,788 of our games.[43] Figure 2 displays a scatter plot of the share of positive game reviews and the corresponding metascores, showing that they are indeed highly correlated.

Figure 2: **Metascore and share of positive reviews**



6.2.2 Alternative estimations

So far, our empirical analysis has been based on a single econometric method only, the control function approach to OLS. To test the robustness of our main results with respect to alternative methods, we additionally run GMM and negative binomial regressions displayed in Table 5. Relative to the CF approach, GMM should be less efficient but more robust (Wooldridge 2015, p. 428). Compared to our main model in Table 4, the standard errors of our GMM estimation indeed are larger. The parameter estimates are, however, quantitatively and qualitatively very similar for both games and apps. Our second alternative estimator is the CF approach to the negative binomial regression model since our dependent variable in principle is a count measure, ranging from 1 to 92 for games where the count data property may be more relevant than for apps where it ranges from 1 to 965. We use the negative binomial model since there is evidence for overdispersion in our data which implies that the negative binomial model is to be preferred over the more standard poisson regression. Our negative binomial regression results are qualitatively no different from our main estimation results, thereby reinforcing our prior findings. The CF terms are statistically

[43]The fact that a metascore is only visible for 5.8 percent of all games (on the Steam page) led us to use consumer ratings instead of professional ratings as our measure of satisfaction in the first place.

highly significant, underscoring the importance of instrumenting our set of endogenous variables. The negative binomial regression model is well suited to account for left-censoring of the dependent variable (Winkelmann 2008, Ch. 6). In our case, the left-censoring is present at the value of 1, which provides additional evidence for the robustness of our results.

Table 5: **Alternative estimators**

	Games		**Apps**	
	GMM Coeff. (std. err.)	**NBREG CF Coeff. (std. err.)**	**GMM Coeff. (std. err.)**	**NBREG CF Coeff. (std. err.)**
Satisfaction	0.059**	0.225***	0.074***	0.227***
	(0.028)	(0.080)	(0.014)	(0.066)
Value	2.164**	7.701***	0.098***	0.248*
	(1.082)	(3.045)	(0.036)	(0.152)
Value^2	-1.027*	-3.635**	-0.004***	0.012**
	(0.541)	(1.516)	(0.002)	(0.006)
Competition	-0.500***	-1.542***	-0.301*	2.639***
	(0.178)	(0.504)	(0.161)	(0.782)
F-test for 1st stage residuals joint significance	—	8.420	—	36.150
(p-value)	—	0.078	—	0.000
Hansen-Sargan J test	4.338	—	17.177	—
(p-value)	0.9589	—	0.143	—

Notes: The dependent variable is the number of titles produced by the own publisher in a given segment. The specifications additionally include controls for release month, release year, and segment. The "first stage residuals" refer to the OLS residuals generated from the first stage regressions of the endogenous variables satisfaction, value, value^2, competition on the exogenous variables and the sets of instruments. Satisfaction is measured as the share of positive ratings for games and as the average rating score on a 1-5 scale for apps. The empirical proxy for segment value in the case of games is price and the number of installations for apps. Competition is measured as the number of competing publishers in the own segment for both apps and games. Satisfaction, value, value^2, and competition as well as the dependent variable are all IHS-transformed. All first stage F-tests are statistically highly significant. Standard errors are bootstrapped. $*\ p < 0.10$, $**\ p < 0.05$, $***\ p < 0.01$.

Our theoretical model predicts a non-monotonic (i.e., concave) relationship between segment value and portfolio size while it predicts monotonic relationships for portfolio size and satisfaction as well as competition. In an additional robustness check, we verify that empirically there is indeed a non-monotonic relationship between segment value and portfolio size alone, but not between portfolio size and the other two variables. To do so, we added squared terms for satisfaction and competition and ran OLS control function regressions. These regressions now contain as many as six endogenous variables. We instrument squared satisfaction and squared competition by the respective squared instruments used in the main specification. While our identification strategy seems to partially work in a statistical sense with high F-test statistics for the first stage and a large Hansen J-test at least for games, our robustness check results should be taken with great caution. Conditional on trusting our identification approach, we do not find evidence for the presence of quadratic effects of satisfaction or competition: For games, both terms on competition are positive. While the coefficient on the linear term of satisfaction is statistically weakly significantly negative and the quadratic term is statistically insignificantly positive, there is a positive relationship between portfolio size and satisfaction for only 2.7 percent of the observations—i.e., for the relevant part of the observations, the relationship between satisfaction and portfolio size is negative as predicted by our theoretical model. Similarly, for apps we find a statistically significantly concave effect of competition and a statistically significantly convex effect of satisfaction. However, over the values for competition and satisfaction observed in our data, the mapping between portfolio size and competition is positive throughout while it is uniquely negative for portfolio size and satisfaction. We hence do not find evidence for quadratic effects of satisfaction or competition either because of a lack of statistical grounds or because values of satisfaction and competition where the effects become non-monotonic, are not observed in our data.

7 Conclusion

This paper studies the interaction between product variety and customer satisfaction, both theoretically and empirically. The relation between these two variables is particularly important in digital markets, as for digitized products the offered variety can be increased at relatively low costs and consumers frequently purchase goods, which makes customer satisfaction an important dimension for a firm's demand.

In our theoretical model, we show that the two strategic variables are substitutes for a firm, as they target different consumer groups. While customer satisfaction mainly aims at retaining existing consumers, a larger product line helps a firm to increase demand from switching consumers. Across firms, the interaction between product variety and customer satisfaction is, however, less clear, and the two variables can be strategic substitutes or complements. We also show how the market environment affects a firm's product variety choice. We find that the value of a segment has a non-monotonic effect on product variety—i.e., there is an inverted u-shaped relationship. Moreover, the number of competitors in the market has a positive effect on product variety. These effects are genuinely due to the interplay between product variety and customer satisfaction, and differ substantially from those obtained in classic oligopoly models.

In our empirical analysis, we test the predictions arising from the theoretical model using data sets from two markets: video games and mobile applications. Both data sets have a lot of heterogeneity in all relevant dimensions and provide us with observable information to either get a direct measure or at least a proxy for our key variables. They also allow us to deal with endogeneity problems by using plausible instruments, such as policy changes and cost shifters. We find a negative relationship between product variety and consumer satisfaction in both markets, which demonstrates that the two variables are (strategic) substitutes. We also find an inverted u-shape relationship between segment value and product variety, while the number of competitors is positively related with product line length. Therefore, there is strong empirical support for the hypotheses from the theoretical model, which emphasizes the relevance of the model's novel effects.

An important managerial implication of our analysis is that the optimal product variety choice interacts with customer satisfaction. Therefore, investments in product lines and in customer satisfaction should not be considered in isolation. Specifically, the relation between the two variables is negative, because customer satisfaction helps to retain consumers and increase the repurchasing rate, whereas a larger product line helps to attract new consumers. Consequently, if, for instance, a manager contemplates about reducing the product line (e.g., to save costs), a profitable complementary strategy could be to increase measures that make consumers more satisfied with their existing products, such as providing better functionality of products or an increased updating frequency.

Another implication is that the optimal adjustment in product variety to changing market conditions can be opposite to first-glance intuition; therefore, such changes should be made with care. Suppose that a market segment becomes more valuable (e.g., because there is a cost reduction). The standard reaction would most likely be an increase in product portfolio size. However, a more profitable response might be to invest in the quality of the existing products while leaving the product line unchanged or reducing it. This strategy is even more valuable if competitors react in the same way and fewer consumers are willing to switch.

We conclude by discussing some avenues for future research, both theoretical and empirical. In the theoretical analysis, we consider a static model to keep the analysis simple (i.e., consumers have previously bought products, but this choice is given). It might be interesting to consider a dynamic model, which explicitly analyzes how the possibility of gaining demand in several periods shapes the incentives to invest in portfolio size and in customer satisfaction. This could provide meaningful insights into the dynamic interplay between the two decisions. Second, we consider a simple demand structure by assuming that products are differentiated in a symmetric way. A richer framework could allow for vertical differentiation between products, implying that some products receive more demand than others.

In the empirical analysis, it would be interesting to study whether our results extend to other (e.g., non-digital) markets, and how they potentially need to be modified. This could provide insights into determining how traditional markets and digital markets differ with respect to product variety choice and the interplay between product variety and customer satisfaction. In addition, in the two markets we consider, consumers repeatedly buy or download apps and games, which is in line with our theoretical model. In other markets, the repurchasing decision occurs less frequently. An interesting question is hence how findings might differ for such markets, which will also help to understand better the interplay between our main strategic variables.

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Appendix

Appendix A: Proofs of Results 1 - 3

Proof of Result 1

We start with part (i) of Result 1. Using the Implicit-Function Theorem to (3) yields

$$\frac{dm_i}{ds_i} = -\frac{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial m_i \partial s_i}}{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial (m_i)^2}}.$$

Due to the fact that $\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\partial (m_i)^2 < 0$—i.e., second-order conditions are satisfied—the sign of dm_i/ds_i is determined by the sign of $\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\partial m_i \partial s_i$. Differentiating $\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\partial m_i$ with respect to s_i, we obtain

$$\operatorname{sign}\left\{\frac{dm_i}{ds_i}\right\} = \operatorname{sign}\left\{-\frac{p_i (v-p_i)^{\frac{1}{\beta}} \left(\sum\limits_{j=1,\, i\neq j}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2}\right\} < 0.$$

Similarly,

$$\frac{ds_i}{dm_i} = -\frac{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial s_i \partial m_i}}{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial (s_i)^2}}.$$

Because $\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\left(\partial s_i \partial m_i\right) = \partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\left(\partial m_i \partial s_i\right)$ and $\partial^2 \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\partial (s_i)^2 < 0$, it follows that $ds_i/dm_i < 0$ As a consequence, m_i and s_i are substitutes for firm i.

We now turn to the part (ii) of Result 1. Applying again the Implicit-Function Theorem to the first-order conditions for m_i to determine how firm i reacts to a change in s_j, we obtain

$$\operatorname{sign}\left\{\frac{dm_i}{ds_j}\right\} = \operatorname{sign}\left\{-\frac{p_i (v-p_i)^{\frac{1}{\beta}} \left(\sum\limits_{j=1,\, i\neq j}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2}\right\} < 0. \tag{8}$$

A higher customer satisfaction level of a rival firm j therefore induces firm i to reduce its product variety. Similarly, we can determine firm i's response in customer satisfaction investment if firm j offers an increased product variety. This yields

$$\operatorname{sign}\left\{\frac{ds_i}{dm_j}\right\} = \operatorname{sign}\left\{\frac{p_i m_i (v-p_i)^{\frac{1}{\beta}} (v-p_j)^{\frac{1}{\beta}}}{M\left(\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2}\right\} > 0. \tag{9}$$

Therefore, a larger product variety of firm j induces firm i to invest more in customer satisfaction. As a consequence, dependent on whether the effect in (8) or the effect in (9) dominates, the two variables are either strategic substitutes or strategic complements.

Proof of Result 2

The symmetric equilibrium is characterized by the three first-order conditions given by (6) and (7). Totally differentiating (6) and (7) of the main text with respect to m^*, s^*, p^*, and v, we obtain

$$-\left(\frac{p^*(M-1)(1-s^*)}{M^2\,(m^*)^2} + f''(m^*)\right)dm^* - \left(\frac{p^*(M-1)}{M^2 m^*}\right)ds^* + \left(\frac{(M-1)(1-s^*)}{M^2 m^*}\right)dp^* = 0,$$

$$-c''(s^*)ds - + \left(\frac{(M-1)}{M^2}\right)dp^* = 0,$$

and

$$\left(-\frac{1}{M\,(m^*)^2} + \frac{p^*(M-1)(1-s^*)}{(v-p^*)\beta M^2\,(m^*)^2}\right)dm^* - \left(\frac{p^*(M-1)}{M^2 m^* \beta (v-p^*)}\right)ds^* -$$
$$-\left(\frac{v(M-1)((1-s^*)}{M^2 m^* \beta (v-p^*)^2}\right)dp^* + \left(\frac{p^*(M-1)(1-s^*)}{M^2 m^* \beta (v-p^*)^2}\right)dv = 0.$$

From these expressions, we can solve for dm^*/dv, ds^*/dv, and dp^*/ds. Doing so and focusing on dm^*/dv (as we are interested in how the equilibrium number of products changes with v), we obtain

$$\frac{dm^*}{dv} = \frac{p^* m^* (M-1)(1-s^*)\left(p^*(M-1) - M^2(1-s^*)c''(s^*)\right)}{M\Big[v\left(p^*(M-1) - c''(s^*)M^2(1-s^*)\right)\left(v\beta - 2\beta p^* + M(n^*)^2 f''(n^*)\right) + \\ + (p^*)^2\left((M-1)\left(\beta p^* - M(n^*)^2 f''(n^*)\right) - M(1-s^*)c''(s^*)\left((M-1)(1-s^*) + \beta M\right)\right)\Big]}. \tag{10}$$

Solving (7) for p^* yields

$$p^* = \frac{v\beta M}{\beta M + (M-1)(1-s^*)}.$$

Inserting the last expression into (10), we obtain

$$\frac{dm^*}{dv} = \frac{(M-1)m^*\beta\mu\left(c''(s^*)M(1-s^*)\mu - v\beta(M-1)\right)}{\left(c''(s^*)M\mu^2 - v\beta\left(M-1\right)^2\right)\left(v\beta(M-1)(1-s^*) + M\left(m^*\right)^2\mu f''(m^*)\right)}, \tag{11}$$

with $\mu \equiv (M-1)(1-s^*) + \beta M > 0$.

The assumption $c''(\cdot) > v\left(M-1\right)^2 / \left(\beta M^3\right)$ implies that the term in the first parentheses of the denominator of the right-hand side of (11)—i.e., $c''(s^*)M\mu^2 - v\beta\left(M-1\right)^2$ is strictly positive. Therefore, the sign of dm^*/dv depends on the sign of the numerator, which is determined by the sign of $c''(s^*)M(1-s^*)\mu - v\beta(M-1)$. This term is positive if and only if

$$v < \frac{c''(s^*)M(1-s^*)\mu}{\beta(M-1)}. \tag{12}$$

For $v \to 0$, this inequality holds, as the left-hand side goes to zero whereas the right hand side is strictly positive. We now turn the opposite case when $v \to \infty$. Then, the left-hand side of (12) tends to infinity. However, because of the assumption $c''(\cdot) > v\left(M-1\right)^2 / \left(\beta M^3\right)$, which ensures an interior solution, $c''(\cdot)$ also tends to infinity. The right-hand side, however, also depends on $1-s^*$. Therefore, to determine whether the right-hand side of (12) also goes to infinity, we need to determine how s^* changes with v. Following the same steps as above, we obtain that ds^*/dv is given by

$$\frac{ds^*}{dv} = \frac{(M-1)\beta\mu}{\left(c''(s^*)M\mu^2 - v\beta\left(M-1\right)^2\right)}. \tag{13}$$

This is strictly positive due the assumption on $c''(\cdot)$. It follows that $s^* \to 1$ as $v \to \infty$. However, this implies that the term $1-s^*$ on the right-hand side of (12) goes to zero. Therefore, for $v \to \infty$, the left-hand side goes to infinity whereas on the right-hand side, one term also goes to infinity but another one goes to zero. This implies that the left-hand side is larger than the right-hand side, that is, the inequality in (12) is reversed for $v \to \infty$. As a consequence, by continuity, we obtain that $dm^*/dv > 0$ for low values of v and $dm^*/dv < 0$ for high values of v.

It remains to show that dm^*/dv changes signs only once. To do so, we now show that there is a unique intersection between the left-hand side of (12)—i.e., v—and the right hand-side of (12)—i.e., $\left[c''(s^*)M(1-s^*)\mu\right] / \left[\beta(M-1)\right]$, as v increases from 0 to ∞. The slope of the left-hand side equals 1. The slope of the right-hand side is given by

$$-\left(\frac{c''(s^*)M\left(\mu + (M-1)(1-s^*)\right)}{\beta(M-1)} - \frac{c'''(s^*)M(1-s^*)\mu}{\beta(M-1)}\right)\frac{ds^*}{dv}.$$

Inserting the value for ds^*/dv and evaluating the slope at the intersection point—i.e., $v = [c''(s^*)M(1-s^*)\mu] \,/\, [\beta(M-1)]$ yields

$$-\frac{c''(s^*)\left(\mu+(M-1)(1-s^*)\right)-c'''(s^*)(1-s^*)\mu}{\beta M c''(s^*)}.$$

Due to the assumption that $c'''(\cdot)$ is either negative or, in case it is positive, small relative to $c''(\cdot)$, this expression is strictly negative. It follows that at any intersection point, the right-hand side crosses the left-hand side with a decreasing slope. As the right-hand side is larger than the left-hand side at $v = 0$, it crosses the left-hand side from above. Hence, there can only be a single intersection point.

Proof of Result 3

In the same way as in the proof of Result 2, we can totally differentiate (6) and (7) with respect to m^*, s^*, p^*, and M, and solve for dm^*/dM, ds^*/dM, and dp^*/dM. Focusing on dm^*/dM and using $p^* = (v\beta M) \,/\, (\beta M + (M-1)(1-s^*))$, we obtain

$$\frac{dm^*}{dM} = \frac{m^*\beta v(M-1)\left(\beta M+(M-1)(1-s^*)\right)\left(v\beta(M-1)+c''(s^*)M(1-s^*)\mu\right)}{\left(c''(s^*)M\mu^2 - v\beta\left(M-1\right)^2\right)\left(v\beta(M-1)(1-s^*)+M\left(m^*\right)^2\mu f''(m^*)\right)}.$$

By the same argument as in the proof of Result 2, the term in the first parentheses in the denominator is strictly positive. As all other terms are strictly positive as well, $dm^*/dM > 0$.

Appendix B: Extensions

In this Appendix, we present several extensions of our model. In Section B.1, we present the analysis of the case without economies of scope with respect to investment into customer satisfaction. In Section B.2, we present the analysis of a model with sequential instead of simultaneous decisions where investments in product variety and customer satisfaction precede the pricing decision. In Section B.3, we present the analysis of the situation in which investment in customer satisfaction also has a positive effect on attracting switching consumers. In Section B.4, we present the analysis of the case of asymmetric firms. In Section B.5, we present a concrete example with specific cost functions, which allows to obtain a closed-form solution. Finally, in Section B.6, we present the analysis of the case in which firms do not charge prices but maximize demand.

B.1: No Economies of Scope

The main model considers a situation in which investment in customer satisfaction applies to all products of a firm. This implies strong economies of scale. In this section, we consider the opposite case—i.e., no economies of scale. This implies that for each of its existing products, a firm decides about the satisfaction level separately.

Denoting by ℓ the product of firm i, with $\ell = 1, ..., m_0$, the overall costs of firm i from investing in customer satisfaction are given by $\sum_{\ell=1}^{m_0} c\left(s_{\ell,i}\right)$, where $s_{\ell i}$ is the customer satisfaction level of firm i's product ℓ. The resulting profit function is then given by

$$\Pi_i(\mathbf{m},\mathbf{s},\mathbf{p}) = \frac{\sum_{\ell=1}^{m_0} s_{\ell,i}}{Mm_0}\left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{\ell=1}^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}}\right)+\frac{\sum_{j=1}^{M}\sum_{\ell=1}^{m_0}(1-s_{\ell,j})}{Mm_0}\left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{j=1}^{M}\sum_{\ell=1}^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}}\right)-fm_i-\sum_{\ell=1}^{m_0} c\left(s_{\ell,i}\right),$$

with $\mathbf{m} = \{m_1, ..., m_M\}$ and $\mathbf{p} = \{p_{11}, ..., p_{m_M,M}\}$, as above, but $\mathbf{s} = \{s_{11}, ..., s_{M,m_0}\}$.

Determining the first-order conditions, using that each firm i optimally chooses the same satisfaction level for each of its existing products m_0 and sets the same price for each of its m_i products it offers in $t = 1$, the first-order conditions are

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial m_i} = \frac{\sum_{j=1}^{M}(1-s_j)p_i\left(v-p_i\right)^{\frac{1}{\beta}}\left(\sum_{j=1,\, i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0,$$

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial s_{\ell,i}} = \frac{p_i}{Mm_0}\left(1-\frac{m_i(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right) - c'(s_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial p_{\ell,i}} = \frac{s_i}{Mm_i} + \frac{\sum_{j=1}^{M}(1-s_j)\left(v-p_i\right)^{\frac{1-\beta}{\beta}}}{M\beta\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}\left(\frac{p_i\left(v-p_i\right)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}} - \left(p_i - \beta\left(v-p_i\right)\right)\right) = 0.$$

It is easy to check that the only difference to the first-order conditions of the main model is that the first-order condition with respect to $s_{\ell,i}$ contains an additional m_0 in the denominator of the first term. However, this does not affect the qualitative results with respect to ds_i/dm_i and ds_i/dm_j, that is $ds_i/dm_i < 0$ and $ds_i/dm_j > 0$. Moreover, as the first-order condition with respect to m_i is the same as in the main model, we also obtain $dm_i/ds_i < 0$ and $dm_i/ds_j < 0$. Therefore, Result 1 is unchanged.

Using symmetry in the first-order conditions and simplifying them in the same way as in Section 3 of the main text yields

$$\frac{(1-s^*)(M-1)p^*}{m^* M^2} - f'(m^*) = 0, \quad \frac{(M-1)p^*}{M^2 m_0} - c'(s^*) = 0,$$

and

$$\frac{M\beta\,(v-p^*) - (M-1)p^*\,(1-s^*)}{m^* M^2 \beta\,(v-p^*)} = 0.$$

Again, these equations are the same as the one of the main model, given by (6) and (7), with the exception that the second equation has an additional m_0 in the denominator of the first term of the left-hand side. Solving for dm^*/dv yields

$$\frac{dm^*}{dv} = \frac{(M-1)m^*\beta\mu\,(c''(s^*)m_0 M(1-s^*)\mu - v\beta(M-1))}{\left(c''(s^*)m_0 M\mu^2 - v\beta\,(M-1)^2\right)\left(v\beta(M-1)(1-s^*) + M\,(m^*)^2\,\mu f''(m^*)\right)},$$

By the same argument as in the proof of Result 2, we can show that $m^\star$ changes again non-monotonically with v—i.e., it is increasing in v for v below a certain threshold, but decreasing in v for v above this threshold. Therefore, Result 2 caries over. Similarly, solving dm^*/dM yields

$$\frac{dm^*}{dM} = \frac{m^*\beta v(M-1)\,(\beta M + (M-1)(1-s^*))\,(v\beta(M-1) + c''(s^*)m_0 M(1-s^*)\mu)}{\left(c''(s^*)m_0 M\mu^2 - v\beta\,(M-1)^2\right)\left(v\beta(M-1)(1-s^*) + M\,(m^*)^2\,\mu f''(m^*)\right)} > 0,$$

which implies that also Result 3 carries over.

B.2: Customer Satisfaction and Product Variety Choices Precede Price Choices

In the main model, we considered the case in which firms simultaneously choose product variety, customer satisfaction, and product prices. In this section, we analyze the case in which the first two variables—i.e., product variety and customer satisfaction—are chosen before product prices are set. A natural reason for such a sequential timing could be that product variety and customer satisfaction are more long-term decisions than product prices. In particular, in some industries product prices can be changed at a relatively fast speed, whereas adding products to the portfolio or improving the functionality of products may take longer.[44]

The sequential game therefore unfolds as follows. In the first, stage, each firm $i = 1, ..., M$

[44]Nevertheless, as mentioned in the main text, in many digital markets, it is relatively simple and takes little time to add or withdraw products, and changes in the software code to improve the functionality can also be introduced at a fast rate. Therefore—and also to bring out our effects in the simplest way—we consider a simultaneous timing in the main model.

chooses the number of its products, m_i, and the customer satisfaction level, s_i. Given these choices, in the second stage, each firm sets prices for its products $p_{\ell,i}$, with $\ell = 1, ..., m_i$. We analyze the game by backward induction and solve for the subgame-perfect equilibrium of the game.

In the second stage, the first-order condition with respect to $p_{\ell,i}$ is the same as in the main model, taking m_i and s_i, $i = 1, ..., M$, from the previous stage as given. The first-order condition is given by (5), which again uses that a firm sets the same prices for its products.

In the first stage, using the Envelope-Theorem, the two first-order conditions for m_i and s_i can be written as

$$\frac{\partial \Pi_i}{\partial m_i} = \left.\frac{\partial \Pi_i}{\partial m_i}\right|_{sim} + \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial \Pi_i}{\partial p_j} \left(- \frac{\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2 \Pi_i}{\partial (p_j)^2}} \right) = 0 \tag{14}$$

and

$$\frac{\partial \Pi_i}{\partial s_i} = \left.\frac{\partial \Pi_i}{\partial s_i}\right|_{sim} + \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial \Pi_i}{\partial p_j} \left(- \frac{\frac{\partial^2 \Pi_j}{\partial p_j \partial s_i}}{\frac{\partial^2 \Pi_i}{\partial (p_j)^2}} \right) = 0, \tag{15}$$

respectively, where we used that $dp_j/dm_i = -\left(\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i}\right) / \left(\frac{\partial^2 \Pi_i}{\partial (p_j)^2}\right)$ and $dp_j/ds_i = -\left(\frac{\partial^2 \Pi_j}{\partial p_j \partial s_i}\right) / \left(\frac{\partial^2 \Pi_i}{\partial (p_j)^2}\right)$. In these equations, $\left.\frac{\partial \Pi_i}{\partial m_i}\right|_{sim}$ and $\left.\frac{\partial \Pi_i}{\partial s_i}\right|_{sim}$ are the respective first-order conditions from the simultaneous game and are given by (3) and (4). In addition to the first-order conditions of the simultaneous game, those of the sequential game also consider the effect that a change in m_i and s_i has on the prices chosen by rival firms in the second stage. This is represented by the second term in the two conditions (14) and (15).

From firm i's profit function, we obtain

$$\frac{\partial \Pi_i}{\partial p_j} = \frac{\sum_{j=1}^{M}(1 - s_j)}{M} \left(\frac{m_i p_i (v - p_i)^{\frac{1}{\beta}} (v - p_j)^{\frac{1-\beta}{\beta}}}{\beta \left(\sum_{j=1}^{M} m_j (v - p_{\ell,j})^{\frac{1}{\beta}} \right)^2} \right) > 0. \tag{16}$$

We now turn to the terms for dp_j/dm_i and dp_j/ds_i. The terms in the respective numerators are the cross derivatives of firm j's profit function with respect to p_j and m_i in (14) and with respect to p_j and s_i in (15). These terms are given by

$$\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i} = -\frac{\sum_{j=1}^{M}(1-s_j)}{M}\left(\frac{(v-p_j)^{\frac{1-\beta}{\beta}}(v-p_i)^{\frac{1}{\beta}}}{\beta\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}\right)\left(2\frac{p_j(v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}} - (p_j - \beta(v-p_j))\right) \tag{17}$$

and

$$\frac{\partial^2 \Pi_j}{\partial p_j \partial s_i} = -\left(\frac{(v-p_j)^{\frac{1-\beta}{\beta}}}{M\beta\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right)\left(\frac{p_j(v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}} - (p_j - \beta(v-p_j))\right) > 0, \tag{18}$$

respectively. While we cannot determine the sign of (17), the sign of (18) is strictly positive. This is due to the fact that the first term in parentheses is positive, while the second term in parentheses is negative. The latter follows from the first-order condition for the prices, as given by (5). This first-order condition can only be satisfied if the following inequality holds:

$$p_j - \beta(v-p_j) > \frac{p_j(v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}. \tag{19}$$

The reason is that the first term in (5)—i.e., $s_j/(Mm_j)$—is strictly positive, which implies that the second term must be negative. This is only true if (19) holds. The implication of this argument is that, in (18), the second term in parentheses is negative; hence, $\partial^2 \Pi_j/(\partial p_j \partial s_i)$ is positive. Finally, turning to (15) again, the denominator of the term in the large parentheses $\partial^2 \Pi_i/\partial(p_j)^2$, which is strictly negative because of the second-order condition. As a consequence, $dp_j/ds_i > 0$ then implies that the second term in (15) is positive.

Taken these results together, it follows that the level of customer satisfaction is larger in the sequential timing as compared to the simultaneous timing. Due to the fact that the second term of (15) is positive, at the point of s_i at which the first-order condition in the simultaneous timing is fulfilled (i.e., the equilibrium value of s_i in the simultaneous timing), the first-order condition in the sequential timing is positive. It follows that the maximum in the sequential timing must lie to the right of the maximum of the simultaneous timing, which implies that the equilibrium value of customer satisfaction is larger in the sequential timing. The intuition is that investing more in customer satisfaction by firm i induces fewer consumers to switch, which implies that competition for switching consumers is reduced.

Hence, the pricing pressure on products is lower. This induces all rival firms to increase their prices, which is beneficial for firm i. Therefore, firm i has a stronger incentive to raise s_i.

Instead, for the optimal number of products, the direction of the change between the sequential and the simultaneous timing is not clear. This is because the sign of $\partial^2\Pi_j/\left(\partial p_j \partial m_i\right)$ is not clear-cut. This sign depends on the sign of the last term in parentheses. In contrast to the term in (18), this term has two times the positive expression $\left(p_j(v-p_j)^{\frac{1}{\beta}}\right) / \left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)$ instead of only once. Therefore the term can either be positive or negative. The intuition is that an increase in m_i has a twofold effect on pricing incentives of rivals. First, each rival will serve fewer consumers with its products, which, similar to the effect outlined in the previous paragraph, induces rivals to increase prices. Second, a larger number of products by firm i enhances competition for switching consumers, which leads to downward pressure in prices. Therefore, the overall effect is ambiguous.

We now turn to the interaction between the number of products and customer satisfaction, within a firm and between firms. To determine these effects, we need to take the derivative of the first-order condition (14) with respect to s_i and s_j, and of the first-order condition (15) with respect to m_j. Unfortunately, the resulting expressions are rather unwieldy without clear results. However, we performed numerous numerical simulations with different functions and verified that in almost all of them, the direction of the interaction between the number of products and customer satisfaction is the same as in Result 1.[45]

We here provide two reasons for the findings that we obtained in our simulations: First, in the first-order conditions (14) and (15), the first term is the same as in the first-order condition for the simultaneous case, which by itself is responsible for Result 1. In the simulations, we obtain that for many parametrizations, the effect resulting from the first term is dominating the effect resulting from the second term. Second, the effect of the second term often also goes in the same direction as that of the first term. To give an example for this, consider the first-order condition for m_i, (14). In the second term of this expression, we have the fraction between $\partial^2\Pi_j/\left(\partial p_j \partial m_i\right)$ and $\partial^2\Pi_i/\partial\left(p_j\right)^2$. In both terms, the levels of customer satisfaction show up only in the term $\sum_{j=1}^{M}(1-s_j)$, which implies that they cancel out in the fraction. Therefore, the derivative of the second term of (14) with respect to s_i and s_j is equal to

$$\sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial^2\Pi_i}{\partial p_j \partial s_i}\left(-\frac{\frac{\partial^2\Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2\Pi_i}{\partial (p_j)^2}}\right) \quad \text{and} \quad \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial^2\Pi_i}{\partial p_j \partial s_j}\left(-\frac{\frac{\partial^2\Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2\Pi_i}{\partial (p_j)^2}}\right),$$

[45]The numerical simulations are available from the authors.

respectively. From (16), it is easy to see that $\partial^2\Pi_i/(\partial p_j \partial s_i) < 0$ and $\partial^2\Pi_i/(\partial p_j \partial s_i) < 0$. Moreover, as explained above, the sign of $\partial^2\Pi_j/(\partial p_j \partial m_i)$ in these expressions is determined by the sign of

$$\beta(v-p_j) - p_j + 2\frac{p_j(v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}. \tag{20}$$

Inspection of (5) shows that for small values of s_i, the first term of (5) is small, which implies that the second term in the large parentheses in (5) must be close to zero, so that (5) holds. This, however, implies that the expression in (20) is positive. Taken together, for small values of customer satisfaction, the derivative of the second term in (14) with respect to s_i and s_j, respectively, is negative, and therefore goes in the same direction as the first term, which amplifies Result 1.

We now turn to Results 2 and 3. In the unique symmetric equilibrium, all firms choose the same number of products, denoted by $m^\star$, and set the same customer satisfaction level, denoted by $s^\star$, in the first stage, and, in the second stage, set the same price $p^\star$ for their products. As the first-order conditions in the second stage are the same as in the simultaneous game, the equation for the equilibrium price is the same as that shown in the proof of Result 2 and is given by

$$p^* = \frac{v\beta M}{\beta M + (M-1)(1-s^*)}.$$

Using it in the two first-order conditions (14) and (15) and simplifying, these first-order conditions are given by

$$\frac{v\beta(1-s^*)(M-1)}{Mm^\star\mu} - f'(m^\star) - \frac{v\beta(1-s^*)\left(s^*(M-1)-1\right)}{M\mu\left(Mm^\star(1+\beta)-2s^*(M-1)\right)} = 0, \tag{21}$$

and

$$\frac{v\beta(M-1)}{M\mu} - c'(s^\star) - \frac{v\beta m^\star s^*(M-1)}{M\mu\left(Mm^\star(1+\beta)-2s^*(M-1)\right)} = 0, \tag{22}$$

where, as above, $\mu \equiv (M-1)(1-s^*) + \beta M$.

Totally differentiating (21) and (22) with respect to m^*, s^*, p^*, and v, is tedious but standard calculations show that the sign of $m^\star/dv$ is given by the sign of

$$c''(s^\star)M(1-s^*)\mu\eta\left(m^\star(\beta M(M-1)+M^2-(M-1)(1-s^*)) - 2s^*(M-1)^2\right) -$$

$$-v\beta(M-1)\Big\{(m^\star)^2\left[\beta^2M^2(M-1)+\beta\left(2s^\star M(M-1)+2M^2(M-1)-M\right)+s^\star(M-1)(2M-s^\star)+\right.$$

$$\left.+(M^2-1)(M-1)\right]-2m^\star(M-1)\left(2\beta s^\star(M-1)+2s^\star(M-1)(M-s^\star)-1\right)+4\left(s^*\right)^2(M-1)^3\Big\}.$$

with $\eta \equiv Mm^{\star}(1+\beta) - 2s^{*}(M-1)$. Following the method of the proof of Result 2, this expression is positive for $v \to 0$, negative for $v \to \infty$, and there is a unique value of $v > 0$ at which it is zero. Therefore, $m^{\star}$ changes non-monotonically with v—i.e., it is increasing in v if v is below a certain level, but decreasing for v above this level. Proceeding in the same way for $m^{\star}/dM$ yields that its sign is strictly positive. Therefore, Results 2 and 3 of the simultaneous model also hold with sequential decisions.

B.3: Customer Satisfaction has Positive Effects on Switching Consumers

In the main text, we considered the situation in which the level of customer satisfaction affects the probability that a consumer of firm i repurchases from firm i, that is, customer satisfaction helps to retain existing consumers. This is consistent with the findings of many papers in the Marketing literature. However, since a firm can achieve a larger customer satisfaction e.g. by upgrades or better functionality, this rises the quality of firms' products and may therefore also affect the demand from switching consumers. In particular, it is conceivable that a larger level of s_i increases the gross utility of consumers; hence, an investment in s_i may also help the firm to attract a larger mass of consumers who are unsatisfied with their previous product.

In this appendix, we consider the above scenario. To model this effect in a simple way, suppose that the gross utility of a consumer who buys a product from firm i is $v + \psi(s_i)$, with $\psi'(s_i) > 0$ and $\psi''(s_i) < 0$. Therefore, a consumer who buys a product from firm i benefits if s_i is larger, but at a decreasing rate.[46] The assumption that $\psi(\cdot)$ is concave also ensures that second-order conditions are satisfied.

Firm i's profit function can then be written as

$$\Pi_i(\mathbf{m}, \mathbf{s}, \mathbf{p}) = \frac{s_i}{M} \left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v + \psi(s_i) - p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{\ell=1}^{m_i} (v + \psi(s_i) - p_{\ell,i})^{\frac{1}{\beta}}} \right) + \frac{\sum_{j=1}^{M}(1-s_j)}{M} \left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v + \psi(s_i) - p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{j=1}^{M}\sum_{\ell=1}^{m_j} (v + \psi(s_j) - p_{\ell,j})^{\frac{1}{\beta}}} \right) - f(m_i) - c(s_i).$$

The resulting the first-order conditions are (using again that each firm i sets the same price

[46]As in the main model, we consider the case full economies of scope. However, the analysis of Appendix B.1 also applies to this case, that is, our qualitative results would be unchanged if there are no economies of scope.

for all of its products)

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial m_i} = \frac{\sum_{j=1}^{M}(1-s_j)p_i\left(v+\psi(s_i)-p_i\right)^{\frac{1}{\beta}}\left(\sum_{j=1,\, i\neq j}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum_{j=1}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0,$$

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial s_i} = \frac{p_i}{M}\left(1-\frac{m_i(v+\psi(s_i)-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}}\right) + \sum_{j=1}^{M}(1-s_j)\times$$

$$\times\frac{m_i p_i \psi'(s_i)(v+\psi(s_i)-p_i)^{\frac{1-\beta}{\beta}}\left(\sum_{j=1,\, i\neq j}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)}{\beta M\left(\sum_{j=1}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)^2} - c'(s_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial p_{\ell,i}} = \frac{s_i}{Mm_i} + \frac{\sum_{j=1}^{M}(1-s_j)\left(v+\psi(s_i)-p_i\right)^{\frac{1-\beta}{\beta}}}{M\beta\left(\sum_{j=1}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)}\times$$

$$\times\left(\frac{p_i\left(v+\psi(s_i)-p_i\right)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}} - \left(p_i-\beta\left(v+\psi(s_i)-p_i\right)\right)\right) = 0.$$

From the first-order conditions for m_i and s_i, we now determine the relation between m_i and s_i, m_i and s_j, and s_i and m_j, as in Result 1 of the main model. Using the first-order condition for m_i, we obtain that the sign of dm_i/ds_i is given by the sign of

$$-\frac{p_i\left(v+\psi(s_i)-p_i\right)^{\frac{1}{\beta}}\left(\sum_{j=1,\, i\neq j}^{M} m_j(v+\psi(s_i)-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}+$$

$$+\sum_{j=1}^{M}(1-s_j)\frac{p_i\psi'(s_i)\sum\limits_{j=1,i\neq j}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\sum\limits_{j=1}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}(v+\psi(s_i)-p_i)^{\frac{1-\beta}{\beta}}}{M\beta\left(\sum\limits_{j=1}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)^4}\times$$

$$\times\left(\sum_{j=1,i\neq j}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}-m_i(v+\psi(s_i)-p_i)^{\frac{1}{\beta}}\right).$$

The term in the first line is equivalent to that in the main model. Instead, the term in the second and third line is new and arises because of $\psi'(s_i)>0$. The sign of this term is determined by the sign of the term in parentheses in the third line. If, for instance, there are more than two firms and firms are symmetric, this term is positive. It follows that the sign of dm_i/ds_i is no longer clear. The reason is that offering a larger product portfolio size becomes more valuable for firm i if s_i increases the value of each product. However, for this effect to dominate the effect of the main model, which is represented by the first line, $\psi'(s_i)>0$ must be sufficiently large. If $\psi'(s_i)$ is rather small, $dm_i/ds_i<0$ as in the main model.

Turning to the relation between m_i and s_j, the sign is given by the sign of

$$-\frac{p_i\,(v-p_i)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,i\neq j}^{M}m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}-$$

$$-\sum_{j=1}^{M}(1-s_j)\frac{p_i\psi'(s_i)(v+\psi(s_i)-p_i)^{\frac{1}{\beta}}m_j(v+\psi(s_j)-p_j)^{\frac{1-\beta}{\beta}}\sum\limits_{j=1}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}}{M\beta\left(\sum\limits_{j=1}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)^4}\times$$

$$\times\left(2\sum_{j=1,i\neq j}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}-\sum_{j=1}^{M}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right).$$

Again, the term in the first line is the same as that in the main model, whereas the new term is that in the second and third line. Its sign depends again on the sign of the term in parentheses in the third line. For symmetric firms, this term is positive, which implies that the second term is negative overall and therefore the entire expression is strictly negative. The effect of the main model regarding dm_i/ds_i is then reinforced. In particular, if a rival firm j increases s_j, this does not only imply that fewer consumers switch, but also that the rival's products become more attractive. Investing in product portfolio size then becomes less profitable for firm i, as the firm can attract fewer consumers with each of its products.

Finally, the sign of ds_i/dm_j is given by the sign of

$$\frac{p_i m_i (v-p_i)^{\frac{1}{\beta}} (v-p_j)^{\frac{1}{\beta}}}{M\left(\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2} -$$

$$-\sum_{j=1}^{M}(1-s_j)\frac{p_i\psi'(s_i)m_i(v+\psi(s_i)-p_i)^{\frac{1-\beta}{\beta}}m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}}{M\beta\left(\sum_{j=1}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right)^3}\times$$

$$\times\left(2\sum_{j=1,\,i\neq j}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}} - \sum_{j=1}^{M} m_j(v+\psi(s_j)-p_j)^{\frac{1}{\beta}}\right).$$

Again, compared to the main model, the term in the second and third line is new and its sign is determined by the sign of the expression in parentheses in the third line. This expression is positive for symmetric firms, which implies that the new term is negative. As the term in the first line is positive, the sign of ds_i/dm_j is now not clear-cut. The reason is as follows: If a rival increases its number of products, fewer switching consumers will buy from firm i. Since investment in s_i now, in addition to retaining consumers, also increases the value of each new product, it might be less profitable to invest in s_i as the new products obtain less demand. However, as above, this effect is dominated by the effect of the main model if $\psi'(s_i)$ is relatively small.

Overall, this analysis shows that Result 1 of the main model carries over to this extension if $\psi'(s_i)$ is rather small. However, even if it was large, there is still a strong indication that the relation between product variety and customer satisfaction is negative. Specifically, the sign of dm_i/ds_i, which is certainly negative in the main model is now no longer clear, but also the sign of ds_i/dm_j, which was certainly positive in the main model, is now no longer clear. However, the term dm_i/ds_j remains negative and is even exacerbated. Therefore, the indication of the relation between product variety and customer satisfaction goes in the same direction as in the main model.

We now turn to the equilibrium of the game. Denoting the equilibrium values again by m^*, s^*, and p^*, as in the main model, the first-order conditions in the unique symmetric equilibrium can be written as

$$\frac{(1-s^*)(M-1)p^*}{m^* M^2} - f'(m^*) = 0, \quad \frac{(M-1)p^*\beta\left(v+\psi\left(s^*\right)-p^*\right)+\psi'\left(s^*\right)\left(1-s^*\right)}{M^2\beta\left(v+\psi\left(s^*\right)-p^*\right)} - c'(s^*) = 0,$$

and

$$\frac{M\beta\left(v+\psi\left(s^{*}\right)-p^{*}\right)-(M-1)p^{*}\left(1-s^{*}\right)}{m^{*}M^{2}\beta\left(v+\psi\left(s^{*}\right)-p^{*}\right)}=0.$$

Following the proof of Result 2 and totally differentiating these first-order conditions with respect to m^{*}, s^{*}, p^{*}, and v, we can determine $dm^{\star}/dv$. Tedious but otherwise routine calculations show that

$$\operatorname{sign}\left\{\frac{dm^{\star}}{dv}\right\}=\operatorname{sign}\left\{\beta n(M-1)\mu\left[c''\left(s^{*}\right)M(1-s^{*})\mu-\beta(M-1)\left(v+\psi\left(s^{*}\right)\right)+\psi''\left(s^{*}\right)\mu(1-s^{*})\right]\right\},$$

with $\mu\equiv(M-1)(1-s^{*})+\beta M$, as above. In the same way as in the proof of Result 2, we can show that this expression is positive if $v\to 0$, negative if $v\to\infty$, and that there is a unique value of v at which it switches signs. Hence, Result 2 of the main model also holds in this extension.

Similarly, totally differentiating these first-order conditions with respect to m^{*}, s^{*}, p^{*}, and M, we can solve for $dm^{\star}/dM$ to get

$$\operatorname{sign}\left\{\frac{dm^{\star}}{dM}\right\}=$$

$$=\operatorname{sign}\Big\{\beta\left(v+\psi\left(s^{*}\right)\right)^{2}(M-1)\mu\xi+(1-s^{*})M\left(v+\psi\left(s^{*}\right)\right)\mu^{2}c''\left(s^{*}\right)-(1-s^{*})M\left(v+\psi\left(s^{*}\right)\right)\xi\mu^{2}\psi''\left(s^{*}\right)$$

$$-\psi'\left(s^{*}\right)(M-1)\Big[\left(v+\psi\left(s^{*}\right)\right)\beta\mu\left((M-1)(1-s^{*})(2M(1-s^{*})-M+2)+\beta M(4(1-s^{*})+M(2s^{*}-1)\right)$$

$$+\psi'\left(s^{*}\right)\mu^{3}(1-s^{*})\Big]\Big\}$$

with $\xi\equiv(M-1)^{2}(1-s^{*})+M(M-2)\beta$. Due to the fact that $\psi''(\cdot)<0$, all terms in the first line of the right-hand side are strictly positive. Instead, the sign of the terms in the second and third line are not clear to rank. However, they are all multiplies of $\psi'\left(s^{*}\right)$. Therefore, if $\psi'\left(\cdot\right)$ is relatively small, the terms in the first line dominates, which implies that $dm^{\star}/dM>0$. Therefore, Result 3 of the main model holds also in this extension if $\psi'\left(\cdot\right)$ is relatively small.

B.4: Asymmetric Firms

In the main model, we considered the case of symmetric firms, that is, all firms offer the same product variety at the outset (i.e., $m_{10}=m_{20}=\cdots=m_{M0}$). We now demonstrate that our results carry over to the case of asymmetric firms. To simplify the exposition, we consider a situation with two different types of firms, where one type of firms has a smaller product variety at the outset than the other type.[47] As will become clear, the situation with

[47]Firms may also differ in other dimensions. For example, they may have different costs to offer an

M different firms can be tackled in the same way and delivers qualitatively the same results.

Suppose that, among the M firms, $K < M$ firms offer a product variety $\underline{m}_0$ at the outset, whereas $M-K$ firms offer a variety of $\overline{m}_0$ at the outset, with $\underline{m}_0 \neq \overline{m}_0$. The profit function of firm i, with $m_{0i} \in \{\underline{m}_0, \overline{m}_0\}$ is then given by

$$\Pi_i(\mathbf{m},\mathbf{s},\mathbf{p}) = \frac{m_{0i}s_i}{K\underline{m}_0 + (M-K)\overline{m}_0}\left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{\ell=1}^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}}\right) + \frac{\sum_{j=1}^{M} m_{0j}(1-s_j)}{K\underline{m}_0 + (M-K)\overline{m}_0}\left(\frac{\sum_{\ell=1}^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum_{j=1}^{M}\sum_{\ell=1}^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}}\right) -$$

$$-f\left(m_i\right) - c\left(s_i\right).$$

The resulting first-order conditions are

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial m_i} = \frac{\sum_{j=1}^{M} m_{0j}(1-s_j)p_i\left(v-p_i\right)^{\frac{1}{\beta}}\left(\sum_{j=1,\, i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{\left(K\underline{m}_0 + (M-K)\overline{m}_0\right)\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0,$$

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial s_i} = \frac{p_i m_{0i}}{K\underline{m}_0 + (M-K)\overline{m}_0}\left(1 - \frac{m_i(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right) - c'(s_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial p_{\ell,i}} = \frac{m_{0i}s_i}{\left(K\underline{m}_0 + (M-K)\overline{m}_0\right)m_i} +$$

$$+\frac{\sum_{j=1}^{M} m_{0j}(1-s_j)\left(v-p_i\right)^{\frac{1-\beta}{\beta}}}{\left(K\underline{m}_0 + (M-K)\overline{m}_0\right)\beta\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}\left(\frac{p_i\left(v-p_i\right)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}} - \left(p_i - \beta(v-p_i)\right)\right) = 0.$$

It is straightforward to check that the signs of the cross-derivatives dm_i/ds_i, dm_i/ds_j, and ds_i/dm_j are the same as in the main model. This is because the terms in the first-order conditions (with the exception of the cost functions that do not affect the cross derivatives) are only multiplied by a different parameter compared to the main model (i.e., $m_{0j}/\left(K\underline{m}_0 + (M-K)\overline{m}_0\right)$ instead of $1/M$), but are not affected otherwise. Therefore, Re-

additional product (i.e. a different $f(\cdot)$-function) or differ in their investment cost of customer satisfaction (i.e., a different $c(\cdot)$-function). Solving for the equilibrium in these situations can be done in the same way as in the case in which firms differ in their initial product variety.

sult 1 also holds with asymmetric firms.

Proceeding in the same way as in the main model, we obtain the equilibrium conditions from these first-order conditions. In the unique symmetric equilibrium, all firms of the same type set the same equilibrium values. Denoting by $\underline{m}^*$, $\underline{s}^*$, and $\underline{p}^*$ the equilibrium levels of firms with a product variety of $\underline{m}_0$ at the outset and by $\overline{m}^*$, $\overline{s}^*$, and $\overline{p}^*$ those of firms with product variety $\overline{m}_0$ at the outset, we can write the equilibrium conditions for the latter three variables as

$$(K\underline{m}_0\,(1-\underline{s}^*)+(M-K)\overline{m}_0\,(1-\overline{s}^*))\times$$

$$\times\frac{\overline{p}^*\,(v-\overline{p}^*)^{\frac{1}{\beta}}\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K-1)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}{(K\underline{m}_0+(M-K)\overline{m}_0)\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)^2}-f'\left(\overline{m}^*\right)=0,$$

$$\frac{\overline{m}_0\overline{p}^*\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K-1)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}{(K\underline{m}_0+(M-K)\overline{m}_0)\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}-c'\left(\overline{s}^*\right)=0,$$

and

$$\frac{\overline{s}^*}{\overline{m}^*\,(K\underline{m}_0+(M-K)\overline{m}_0)}-\left(K\underline{m}_0\,(1-\underline{s}^*)+(M-K)\overline{m}_0\,(1-\overline{s}^*)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)\times$$

$$\times\frac{(v-\overline{p}^*)^{\frac{1}{\beta}}\,\overline{m}^*\left((M-K-1)\overline{p}^*-\beta(M-K)\,(v-\overline{p}^*)\right)+\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}K\underline{m}^*\left(\overline{p}^*-\beta\,(v-\overline{p}^*)\right)}{\beta\,(v-\overline{p}^*)\,(K\underline{m}_0+(M-K)\overline{m}_0)\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)^2}-f'\left(\overline{m}^*\right)=0.$$

In a similar way, we can write the equilibrium conditions for $\underline{m}^*$, $\underline{s}^*$, and $\underline{p}^*$. This provides us with six equations for the six equilibrium values.

Proceeding in the same way as in the proofs of Results 2 and 3, we can totally differentiate these equations with respect to the six equilibrium variables as well as v and M, which allows us to determine $d\overline{m}^*/dv$, $d\underline{m}^*/dv$, $d\overline{m}^*/dM$, and $d\underline{m}^*/dM$. The resulting expressions are very long compared to the main model due to the additional parameters $\underline{m}_0$, $\overline{m}_0$, and K. However, we can show that the results are akin to those of the main model. In particular, $d\overline{m}^*$ and $d\underline{m}^*$ change non-monotonically with v, that is, $d\overline{m}^*/dv$ and $d\underline{m}^*/dv$ are both positive for small values of v, but negative for large values of v and there is a unique value of v at which both derivatives are zero. Instead, $d\overline{m}^*/dM$, and $d\underline{m}^*/dM$ are positive for all values of v. Therefore, Results 2 and 3 also hold with asymmetric firms.

Finally, we note that considering more than two types of firms leads to similar results. The model becomes more complicated to solve, as, given that there then are k different types of firms, with $2\leq k\leq M$, there are $3\times k$ unknowns. However, the method is the same and the results are qualitatively similar.

B.5: Concrete Example

In this appendix, we provide a concrete example that allows for closed-form solutions. Consider the following functional forms for the firms' cost functions: $f(m_i) = fm_i$ and $c(s_i) = cs_i^2$, that is, marginal costs for adding a new product are constant and marginal costs for investment in customer satisfaction are increasing. These assumptions seem reasonable in many industries: once a firm has entered a market segment (and incurred the respective fixed costs), the cost for launching an additional product in this segment is usually independent of the number of products. Instead, raising customer satisfaction becomes increasingly costly as different instruments to do so have different costs (e.g., providing an update is usually cheaper than general improvements in product quality).

With this formulation, the three first-order conditions are given by

$$\frac{(1-s^*)(M-1)p^*}{m^*M^2} - f = 0, \quad \frac{(M-1)p^*}{M^2} - 2cs^* = 0,$$

and

$$\frac{M\beta(v-p^*) - (M-1)p^*(1-s^*)}{m^*M^2\beta(v-p^*)} = 0.$$

Solving the third first-order condition for p^* yields

$$p^* = \frac{M\beta v}{(M-1)(1-s^*) + \beta M}. \tag{23}$$

Inserting this into the first first-order condition and solving for m^*, we obtain

$$m^* = \frac{\beta v((M-1)(1-s^*))}{fM((M-1)(1-s^*) + \beta M)}. \tag{24}$$

Inserting (23) and (24) into the second first-order condition and solving for s^*, we obtain two solutions:

$$\frac{cM(M-1+\beta M) + \sqrt{\psi}}{2cM(M-1)} \quad \text{and} \quad \frac{cM(M-1+\beta M) - \sqrt{\psi}}{2cM(M-1)},$$

with $\psi \equiv cM\left(cM(M-1+\beta M)^2 - 2v\beta(M-1)^2\right)$. It is easy to check that only the second solution is in the admissible range as the first solution is above 1 for all admissible values, which is not possible due to the fact that s^* is a probability. Inserting the resulting solution for s^* back into the equations for m^* and p^*, the resulting equilibrium expressions for the three variables are

$$m^* = \frac{v\beta\left(\sqrt{\psi} + cM\left(M - 1 - \beta M\right)\right)}{fM\left(cM\left(M - 1 + \beta M\right) + \sqrt{\psi}\right)}, \quad s^* = \frac{cM\left(M - 1 + \beta M\right) - \sqrt{\psi}}{2cM(M-1)}, \tag{25}$$

and

$$p^* = \frac{2cM^2\beta v}{cM\left(M - 1 + \beta M\right) + \sqrt{\psi}}.$$

Taking the derivative of m^* with respect to v yields that it is positive if and only if

$$v < \frac{cM\left(M - 1 + 2\beta M\right)}{2\beta(M-1)},$$

thereby confirming Result 2.

Finally, the sign of the derivative of m^* with respect to M is given by the sign of

$$2c^2M^2\left[cM\left(M(1+\beta) - 1\right)\left(M(M-1)(1+\beta) - 1\right) - v(M-1)\beta\left(M(M-2)(1+\beta) + 2\right)\right] +$$

$$+c^2M^2\left(2M(1+\beta) + M^2(1-\beta^2) - 1\right)\sqrt{\psi} + (\psi)^{\frac{3}{2}},$$

which is strictly positive, due to the assumption that $c > v\left(M-1\right)^2/\left(2\beta M^3\right)$—i.e., the assumption that guarantees an interior solution. This confirms Result 3.

B.6: Firms Maximize Demand

In markets such as the one for mobile apps, only few firms (developers) charge a price for their products. Instead of maximizing profits, firms maximize demand. This either occurs because they monetize their products via advertisements or because they receive a non-monetary benefit from increasing their user base. In this appendix, we show that our results also hold in such a version of the model.

If there are no prices and firms maximize demand, a more valuable market segment is a segment that is characterized by a larger demand. Denoting the mass of consumers by λ, the objective function of firm i can be written as

$$\Pi_i(\mathbf{m}, \mathbf{s}) = \lambda\frac{s_i}{M} + \lambda\left(\frac{\sum_{j=1}^{M}(1 - s_j)}{M}\right)\left(\frac{m_i}{\sum_{j=1}^{M} m_j}\right) - f\left(m_i\right) - c\left(s_i\right).$$

This objective function follows the same lines as that in the main model, but is adjusted for the fact that firms cannot charge prices. The first term is the demand from retained consumers (i.e., consumers who are satisfied with a product of firm i and therefore buy again), whereas the second term is the demand from unsatisfied consumers who decide in favor of s product of firm i. The third and the fourth term are the costs for investing in product variety and customer satisfaction, respectively.

The first-order conditions with respect to m_i and s_i are, respectively,

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{s})}{\partial m_i} = \left(\frac{\lambda \sum_{j=1}^{M} (1 - s_j)}{M} \right) \left(\frac{\sum_{j=1}^{M} m_j - m_i}{\left(\sum_{j=1}^{M} m_j \right)^2} \right) - f'(m_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{s})}{\partial s_i} = \frac{\lambda}{M} \left(1 - \frac{m_i}{\sum_{j=1}^{M} m_j} \right) - c'(s_i) = 0.$$

It is straightforward to verify from the first equation that $dm_i/ds_i < 0$ and $dm_i/ds_j < 0$, whereas the second equation implies $ds_i/dm_i < 0$ and $ds_i/dm_j > 0$. Hence, Result 1 carries over.

In the unique symmetric equilibrium, all firms choose the same number of products, denoted by m^*, and set the same level of customer satisfaction, denoted by s^*. From the two first-order conditions, the resulting equilibrium values are characterized by the following two equations:

$$\lambda \frac{M - 1}{M^2} - c'(s^*) = 0 \quad \text{and} \quad \lambda \frac{M\,(1 - s^*)}{m^* M^3} - f'(m^*) = 0. \tag{26}$$

Following the same method as in the proof of Result 2, we can totally differentiate the two equations with respect to m^*, s^*, and λ and solve for $dm^*/d\lambda$ and $ds^*/d\lambda$. Focusing on $dm^*/d\lambda$, we obtain

$$\frac{dm^*}{d\lambda} = \frac{m^* \left(M^2 (M-1)\,(1 - s^*)\, c''(s^*) - \lambda (M-1)^2 \right)}{M^2 c''(s^*) \left(\lambda (M-1)\,(1 - s^*) + M^2\,(m^*)^2 f''(m^*) \right)}.$$

It follows that $dm^*/d\lambda > 0$ if and only if

$$\lambda < \frac{M^2 (M-1)\,(1 - s^*)\, c''(s^*)}{(M-1)^2}.$$

In the same way as in the proof of Result 2, we can show that the last inequality holds for $\lambda \to 0$, is reversed for $\lambda \to \infty$, and there is a unique intersection point between the left-hand side and the right-hand side as λ goes from 0 to infinity. It follows that $dm^*/d\lambda$ is positive for small values of λ—i.e., the number of products is increasing in the size of the market segment if the segment's size is relatively small—but is negative for high values of λ—i.e., the number of products is decreasing in the size of the market segment for segments with high demand. Therefore, Result 2 of the main model is also valid in this extension.

Proceeding in the same way to determine dm^*/dM, we obtain

$$\frac{dm^*}{dM} = \frac{\lambda(M-2)m^*\left(\lambda(M-1) + M^2\left(1-s^*\right)c''(s^*)\right)}{M^3c''(s^*)\left(\lambda(M-1)\left(1-s^*\right) + M^2\left(m^*\right)^2 f''(m^*)\right)},$$

which is strictly positive. Therefore, a larger number of firms leads to a larger product variety in equilibrium, which is akin to Result 3 of the main model.

Appendix C: Details on Industry Background and Instrumental Variables

C.1: Industry Background

Addressing Survivorship Bias: Having data from one period in time may lead to missing firms (and products) from the past that exited until that date. However, for both markets the costs to stay in the market are very low as there are, e.g., no recurring store fees or maintenance costs. One can see this, for example, by comparing video games with a release date from 2014 in our data (1,522) to the total of video games published on Steam in 2014 (1,772).[48] This means that the majority of video games from 2014 (about 85 percent) are still in the market, which is qualitatively the same for other years and is, in general, also applicable for mobile apps. Of course, there are policy shocks affecting the market environment by removals of firms or increasing costs/decreasing revenues through new policies, thus creating larger exits. In fact, this is our motivation laid out in our identification strategy as described in Section 5. Given the low costs to stay in the market, one can presume that only very low-value video games and apps to leave the market. These games and apps do not play a crucial role for the market. Finally, we also account for year-specific effects by including release year fixed effects in our regressional analyses.

Developers vs. Publishers: An important distinction is the one between developers and publishers in digital markets, especially for video games where the development and distribution is more expensive. A (too) simple way of describing the roles would be that a

[48]See `https://www.polygon.com/platform/amp/2016/12/1/13807904/steam-releases-2016-growth`.

developer programs a software, while the publisher is in charge of selling it. However, the actual relationship between the two parties is more complicated and the degree of influence varies. Following the classification on Steam, we take the field ‘Publisher’ on each page of a video game to identify a producer. In case this information is missing, we take the field ‘Developer’ (245 cases). For apps, we use the developer name given on each app’s Play Store page to identify firms.

C.2: Instrumental Variables

Review Policies on Google’s Play Store: We use the timing of three major announcements by Google revolving around the identification of fake reviews and the removal thereof during our observation period:

- October, 2016: Google announces new tools to combat manipulation attempts like fake reviews and incentivized ratings (`https://android-developers.googleblog.com/2016/10/keeping-the-play-store-trusted-fighting-fraud-and-spam-installs.html`)
- November, 2016: Google announces improved ways to identify and remove fake reviews and ratings, as they should come from genuine users and not be manipulated in any kind (`https://android-developers.googleblog.com/2016/11/keeping-it-real-improving-reviews-and-ratings-in-google-play.html`)
- December, 2018: Google announces enforcement of policy violations in ratings and reviews, e.g., announcing the removal of millions of reviews and ratings within a week (`https://android-developers.googleblog.com/2018/12/in-reviews-we-trust-making-google-play.html`)

Delistings by Steam: We exploit the following three major interventions by Steam leading to a delisting of over 200 video games from the store:

- September, 2016: 31 games by the developer ‘Digital Homicide Games’ were removed because of hostile communications to Steam customers (`https://www.pcgamer.com/valve-removes-digital-homicides-games-from-steam/`)
- September, 2017: 173 games from the publisher ‘Silicon Echo Studios Games’ were removed because of creating many fake games, i.e., almost identical games based on pre-made assets released in quick succession (`https://delistedgames.com/valve-removes-173-fake-steam-games-from-zonitronsilicon-echo/`)

- February, 2018: 17 games from the publisher 'Insel Games' were removed because of manipulation of reviews by the chief executive officer (`https://mmofallout.com/whatever-happened-to-insel-games/`)

We infer the release data and genre of the removed apps from SteamDB to know which of our segments were affected by the delistings.